Ruderman–Kittel–Kasuya–Yosida interaction between zero-dimensional and one-dimensional ferromagnetic inclusions in a matrix of nonmagnetic metal

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Work is devoted to the calculation of Ruderman–Kittel–Kasuya–Yosida interaction between inclusions of spherical shape (zero-dimensional) and whiskerical shape (one-dimensional). The conservation of oscillating character of the interaction independently of ferromagnetic particle sizes is shown.

Nowadays, research of low-dimensional magnetics has become very popular among solid-state physicists. Two-dimensional systems (superthin films and superlattices consisting of different magnetic and nonmagnetic materials) are best investigated. Effective exchange interaction between magnetic layers through nonmagnetic layers was discovered.[1] Now it is considered that this is due to the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction, i.e., free electrons of nonmagnetic layers. Theoretical calculations of this RKKY interaction between magnetic layers were performed.[2] Evidently, the model of the spherical Fermi surface used does not predict the existence of the long periodical (12–24 Å) oscillations of the exchange coefficient, as it follows from the experimental data,[3] but it is a good basis for more complex models, which already take into consideration the nonsphericity of the Fermi surface or the discrete structure of the substance.[4,5]

As for the structures of more lower dimensions, they are studied not so thoroughly, but now they are attracting growing attention because of the potential technical possibility of their preparing. Obviously, the effective exchange interaction between separate inclusions must exist in systems of such objects and it may lead to the availability of new interesting properties. Theoretical calculations of such an exchange interaction between separate small ferromagnetic spheres (zero-dimensional particles), and ferromagnetic whiskers (one-dimensional) through nonmagnetic materials are presented in this letter. The model of the RKKY interaction in the free electrons approximation (spherical Fermi surface) has been used as a basis.

The interaction between two localized magnetic moments through the gas of free electrons is characterized by the energy of the interaction, which has the form[6]

$$E(r)=\frac{4J^2m^*k_f^4}{(2\pi)^3\hbar^2}F(2k_fr)S_1S_2, \qquad (1)$$

where

$$F(t)=\frac{t\cos t-\sin t}{t^4}. \qquad (2)$$

Here, m^* is effective mass of electrons, S_1, S_2 are localized spins, J is the exchange interaction coefficient between localized spins and free electrons [the energy of such an interaction $E=-JS_1s(r)$, where $s(r)$ is a spin density of the free electron gas in the point of localized spin]. Let us consider that a ferromagnetic sphere (whisker) consists of similar spins coupled with the strong right exchange interaction. At first, we find the energy of the interaction between the ferromagnetic sphere of a definite radius and the localized spin situated at a fixed point by integrating the exchange energy over the sphere volume. The integration is the simplest in a spherical coordinate system, where the center coincides with the point of the localized spin and the polar axis goes through the sphere center. The dimensionless variable of the length ($\tilde{R}=2k_fR$ is the dimensionless distance between the sphere center and the localized spin, $\tilde{r}_0=2k_fr_0$ is the radius of the sphere) is more convenient in our case and we shall use it below. Thus, we obtain

$$E=\frac{4J^2m^*k_f}{(2\pi)^3\hbar^2}K(\tilde{R})M_sS, \qquad (3)$$

where S is the localized spin and M_s is the magnetization of the sphere;

$$K(R)=\int_0^{2\pi}d\varphi\int_{\tilde{R}-\tilde{r}_0}^{\tilde{R}+\tilde{r}_0}dx\int_0^{\left(\frac{\tilde{x}+\tilde{R}^2-\tilde{r}_0^2}{2\tilde{R}x}\right)}x^2F(x)\sin(\vartheta)d\vartheta, \qquad (4)$$

where x, φ, and ϑ are variations of the integrating (see Fig. 1). After integrating expression (4) takes the form of

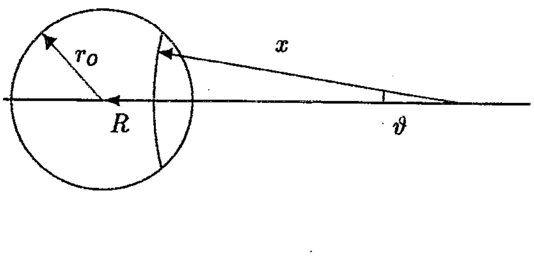

FIG. 1. Scheme of the integration over ferromagnetic sphere.

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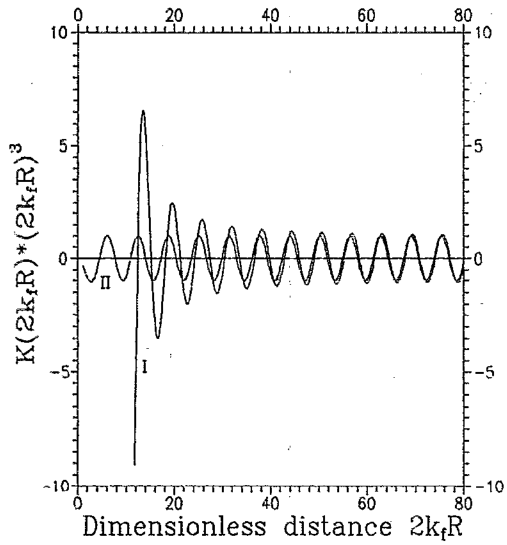

FIG. 2. Distance dependence of the energy of the interaction between ferromagnetic sphere ($\tilde{R}=10$) and localized spin (I) and between two localized spins (II). The effective spin of the ferromagnetic sphere considered to be equal to the value of the localized spins.

$$K(\tilde{R})=\pi\left[\frac{\tilde{R}^2+\tilde{r}_0^2}{\tilde{R}(\tilde{R}^2-\tilde{r}_0^2)}\cos\tilde{R}\,\sin\tilde{r}_0\,\sin\tilde{R}\,\sin\tilde{r}_0\right.$$

$$-\frac{\tilde{r}_0}{\tilde{R}}\cos\tilde{R}\,\cos\tilde{r}_0-\frac{2\tilde{r}_0}{\tilde{R}^2-\tilde{r}_0^2}\sin\tilde{R}\,\cos\tilde{r}_0$$

$$\left.+\left(1+\frac{\tilde{R}^2-\tilde{r}_0^2}{2}\right)\frac{\mathrm{si}(\tilde{R}+\tilde{r}_0)-\mathrm{si}(\tilde{R}-\tilde{r}_0)}{\tilde{R}}\right]. \qquad (5)$$

This is plotted in Fig. 2. Here si(x) is a sine integral determined as $\mathrm{si}(x)=\int_\infty^x \sin t/t\,dt$. If $\tilde{r}_0\rightarrow 0$, then formula (2), which corresponds to the interaction between two localized spins, is obtained in the third-order approximation.

Expression (5) becomes simpler in the approximation of $\tilde{R}\gg\tilde{r}_0$, i.e., for the distances greatly larger than the sphere size. In that case it is obtained that

$$K(\tilde{R})=4\pi(\sin\tilde{r}_0-\tilde{r}_0\cos\tilde{r}_0)\frac{\cos\tilde{R}}{\tilde{R}^3}. \qquad (6)$$

It is interesting to note that for the long distances the "phase" of the interaction (i.e., the form of the dependence of the interaction on distance) does not depend on the sphere size. The interaction has alternating-sign character in the dependence of the distance with any sphere size, even if it is greater than k_f^{-1}. Besides, this dependence is similar to the case of the RKKY interaction between two localized spins. So the ferromagnetic sphere may be substituted with the effective localized spin placed at the sphere center. This effective spin is given by

$$S_{ef}=\frac{4\pi(\sin\tilde{r}_0-\tilde{r}_0\cos\tilde{r}_0)}{k_f^3}M_s. \qquad (7)$$

Evidently, the value and sign of the effective spin depends on the sphere radius. We must mention that for the small distances [as the exact expression (5) shows, see Fig. 2] the energy of interaction between two localized spins is less than the one of the interaction between localized spin and small ferromagnetic sphere with the magnetization corresponding to the same value of the effective spin for long distances.

It is necessary to integrate expression (5) once again over the volume of the second sphere to calculate the energy of two ferromagnetic spheres interaction. The obtained expression is not given here because it is too cumbersome. The energy of the interaction of two spheres becomes simpler if the distance between spheres is much greater than their radii. It is possible to substitute them with the effective spins (7) in that case, and the energy of the interaction takes the form

$$E=\frac{J^2m^*k_j^2}{16(2\pi)^9\hbar^2}K_2(\tilde{R})M_sM_s,$$

$$K_2(\tilde{R})=\frac{16\pi^2\cos\tilde{R}}{\tilde{R}^3}(\sin\tilde{r}_1-\tilde{r}_1\cos\tilde{r}_1)$$

$$\times(\sin\tilde{r}_2-\tilde{r}_2\cos\tilde{r}_2). \qquad (8)$$

The energy decreases as $1/R^3$ as in the case of localized spins. The interaction has oscillating and alternating-sign character in the dependence of the distance independently of the sphere size.

A similar problem was solved for two infinitely thin whiskers, but in that case the solution was found only for long distances between them ($\tilde{R}=2\pi k_fR\gg 1$).

The order of the solution is the same in the previous case. Let us obtain the energy of interaction between the whisker and the localized spin at first:

$$E=\frac{4J^2m^*k_f^3}{(2\pi)^3\hbar^2}L(\tilde{R})SM_s. \qquad (9)$$

Here

$$L(\tilde{R})=2\int_0^\infty F(\sqrt{\tilde{R}^2+x^2})dx,$$

where $F(t)$ corresponds to Eq. (2). Taking into account the condition $\tilde{R}\gg 1$ and introducing a new variable $\tilde{R}ch\phi=\sqrt{x^2+\tilde{R}^2}$ we use the method of stationary phase [$ch^2\phi$ is a slowly alternating function in comparison with cos ($\tilde{R}ch\phi$) if $R\gg 1$]:

$$L(\tilde{R})=2\int_{\tilde{R}}^\infty\frac{\cos(\tilde{R}ch\phi)}{\tilde{R}^2ch^2\phi}d\phi\approx\frac{2}{\tilde{R}^2}\int_{\tilde{R}}^\infty\frac{\cos(\tilde{R}ch\phi)}{ch^2(0)}d\phi$$

$$=-\frac{\pi}{\tilde{R}^2}Y_0(\tilde{R})\approx.$$

$Y_0(\tilde{R})$ is a Bessel function of the second type:

$$\approx-\frac{\pi}{\tilde{R}^2}\sqrt{\frac{2}{\pi\tilde{R}}}\sin\left(\tilde{R}-\frac{\pi}{4}\right)=\frac{\sqrt{\pi}}{\tilde{R}^{5/2}}(\cos\tilde{R}-\sin\tilde{R}), \qquad (10)$$

so the interaction preserves its alternating-sign character in this case, too. The linear density of the energy of the interaction between two parallel whiskers has the similar form (taking into account the substitution S by M_w) due to its translational symmetry.

Now, let us observe the changing low-power decrease of the interaction energy, depending on the distance between the objects of different dimensions. $E(r) \sim 1/r^3$ the zero-dimensional objects (localized spins and small ferromagnetic spheres), $E(r) \sim 1/r^{5/2}$ for one-dimensional ferromagnetic whiskers, and $E(r) \sim 1/r^2$ for two-dimensional ferromagnetic layers.[2] The power of the denominator decreases by $\frac{1}{2}$ when the dimension of the object decreases by 1.

Thus, the calculation of the RKKY interaction (1) between small ferromagnetic inclusions (in the approximation of their spherical shape) and (2) between thin ferromagnetic whiskers (in the approximation of their one-dimension) is done in our work. Preservation of the alternating-sign character of the interaction in both cases in shown. The dependence of the interaction on the ferromagnetic sphere radius is shown, so, ferromagnetic or antiferromagnetic order can, in principle, exist in the systems of such objects.

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