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Influence of thermal fluctuations on time characteristics of a single Josephson element with high damping Exact solution

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Abstract

The present paper presents an analysis of the influence of thermal fluctuations on the superconductive state life time and the turn-on delay time for a single Josephson element with high damping using the obtained exact representation of these time characteristics. The shape of the input pulse is assumed to be rectangular and the duration of the pulse is assumed to be much longer than the turn-on delay of the Josephson element. The analysis has been made within the well-known resistive model of a Josephson junction. It is shown that fluctuations may both decrease and increase the turn-on delay time. The results are presented in a form suitable for comparison with experiment.

Keywords: Fluctuation effects; Josephson effect; Josephson junction

1. Introduction

The investigation of the nonlinear properties of Josephson junctions (JJ's) is very important owing to their broad applications in logic devices. Recently, a lot of attention was paid to Josephson logic devices with high damping [1-6] because of their high-speed switching in comparison with underdamped devices. The papers of Refs. [3] and [4] present a description and analysis of the entire system of logic elements (single flux quantum logic devices). The processes going on in such devices are based on the reproduction of quantum pulses due to spasmodic changing by 2π of the phase difference of overdamped JJ's.

The employment of high-T_c JJ's creates many new problems, one of which is the effect of thermal fluctuations on the speed of logic-device switching and thermally induced errors [5].

Let us note, that the analysis of the influence of thermal fluctuations on different characteristics of JJ's (such as the current-voltage characteristic [7] or the life time of the superconductive state [8]) has been made before on the basis of the Langevin approach. However, the influence of thermal fluctuations on the turn-on delay has not been investigated because of difficulties in obtaining the nonstationary solution of the Fokker-Planck

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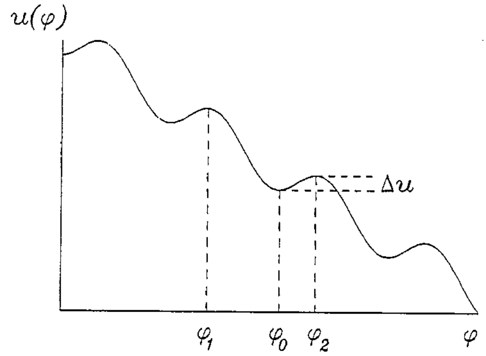

Fig. 1. The example of the potential profile $u(\varphi) = 1 - \cos\varphi - i\varphi$ for $i = 0.5$.

equation, though fluctuations which are present in systems of many JJ's [4] may cause malfunctioning because of the accumulation of errors. At the same time, the noise properties of systems consisting of many elements may be well understood from the noise properties of a single JJ [9].

This paper presents the analysis of the influence of thermal fluctuations on the superconductive state life time and the turn-on delay time for a single Josephson element. The shape of the input pulse is assumed to be rectangular and the duration of the pulse is assumed to be much longer than the turn-on delay of the Josephson element. The analysis has been made within the well-known resistive model of a Josephson junction [10,11]. It is necessary to point out that similar investigations have been already made in Ref. [12] within a piece-wise parabolic approximation of the tilted cosinusoidal potential profile. The present paper, however, demonstrates such an analysis based on the exact time characteristics obtained for the tilted cosinusoidal profile.

2. General equations and set up of the problem

It is known, that processes going on in a single JJ of small size under a current I with fluctuations taken into account are well described by the Langevin equation. Let us restrict our consideration to JJ's with high damping, $\beta \ll 1$, which are widely used in logic elements with high-speed switching. Here $\beta = (2e/\hbar) I_c R_N^2 C$ is the McCamber–Stewart parameter, I_c is the critical current, $R_N^{-1} = G_N$ is the normal conductivity of the JJ, C is the capacitance, e is the electron charge and $\hbar$ is the Planck constant. In this case the Langevin equation takes the following form:

$$\omega_c^{-1} \frac{d\varphi(t)}{dt} = -\frac{du(\varphi)}{d\varphi} - i_F(t), \tag{1}$$

where

$$u(\varphi) = 1 - \cos\varphi - i\varphi \tag{2}$$

is the dimensionless potential profile (see Fig. 1), φ is the difference in the phases of the order parameter on opposite sides of the junction, $i = I/I_c$, $i_F(t) = I_F/I_c$, I_F is the random component of the current, and $\omega_c = 2eR_N I_c/\hbar$ is the characteristic frequency of the JJ. In the case when only thermal fluctuations are taken into account [13], the random current may be represented by white Gaussian noise:

$$\langle i_F(t) \rangle = 0, \quad \langle i_F(t) i_F(t+\tau) \rangle = \frac{2\gamma}{\omega_c}\delta(\tau),$$

where $\gamma = 2ekT/\hbar I_c = I_T/I_c$ is the dimensionless intensity of the fluctuations, T is the temperature and k is the Boltzmann constant.

If the fluctuations are small enough, we can neglect it and Eq. (1) transforms to the dynamic equation

$$\omega_c^{-1}\frac{d\varphi(t)}{dt} = -\frac{du(\varphi)}{d\varphi}. \tag{3}$$

Let us consider a single JJ, biased by a constant current I_0 [3]. If the current going across the junction is not so large, $I_0 < I_c$, then Eq. (3) has the set of steady-state solutions

$$\varphi_n = \arcsin(I_0/I_c) + 2\pi n, \qquad n = 0, \pm 1, \pm 2, \ldots$$

Any such a solution describes a "superconductive" or a "steady-state" S state of the JJ: if $I_0 < I_c$ the voltage across the junction is equal to zero. It is obvious, that if there are no fluctuations in the system, the phase difference will for an infinitely long time be located in the potential minimum number n and without any restrictions we can assume that $n_{\text{initial}} = 0$. However, if the fluctuation intensity is not equal to zero, there is a finite probability for the phase-difference jump across the barrier towards a neighbor potential minimum. The mean time between two sequential jumps is called the life time of the S state.

If the current across the JJ is larger than the critical current, $I > I_c$, there are no steady-state solutions for Eq. (3) and such a state is known as the resistive state (R state).

If a current $I > I_c$ is applied, the JJ switches to the R state. However, an output pulse will be generated not at the same moment, but at a later time. Such a time is called the turn-on delay time between input and output (reproduced) pulses [3,6]. Thus, the input pulse at the time $t = 0$ changes the current across the junction from the value $I_0 < I_c$ to the value $I > I_c$, turns-on the system from the superconductive state to the resistive one and the phase difference begins to slide down in the potential profile, causing the process of Josephson generation.

If the duration of the pulse front is much smaller than the turn-on delay (a rectangular pulse) then at the initial instant of time the phase difference will still be located at the point, where it was in the S state and no generation would be observed. In this case the time of the turn-on delay like the life time of the S state may be defined as the transition time of the phase difference from the point φ_0 (a minimum of the initial potential well) to some point φ over a barrier. Such an identity of time definitions is connected with the fact that in the presence of fluctuations the distinction between terms of the S state and the R state makes no sense, because Josephson generation becomes possible at $I < I_c$ [7]. That is why it is meaningful to introduce a unified term for the life time of the metastable state, τ, implying the life time of the S state at $I < I_c$ and the turn-on delay time at $I > I_c$. In Ref. [6] τ was defined as the time of reaching the point $\pi/2 + 1/2$ from the point $\pi/2$. We followed this procedure, for unification change of $\pi/2 + 1/2$ to π and then τ, obtained analytically in the paper of Ref. [6] without any fluctuations, takes the form ($i > 1$)

$$\tau_d = \frac{1}{\omega_c}\left(\frac{1}{\sqrt{i^2-1}}\left[\pi - 2\arctan\left(\frac{i-1}{\sqrt{i^2-1}}\right)\right]\right). \tag{4}$$

The use of high-T_c superconductors is connected with an increasing intensity of the fluctuations, because the parameter $I_T = 2ekT/\hbar$ increases with increasing temperature T. Proportional increasing of I_c while holding $\gamma = I_T/I_c$ constant causes an increasing of the dissipated power. In the case when γ increases, ignoring of the fluctuations becomes incorrect and it is necessary to operate directly with Eq. (1), so the phase difference gets a random value described by the probability density $W(\varphi, t)$.

It is well known that the Fokker–Planck equation (FPE) for the probability density $W(\varphi, t)$ corresponding to Eq. (1) has the form

$$\frac{\partial W(\varphi,t)}{\partial t} = -\frac{\partial G(\varphi,t)}{\partial\varphi} = \omega_c\frac{\partial}{\partial\varphi}\left\{\frac{du(\varphi)}{d\varphi}W(\varphi,t) + \gamma\frac{\partial W(\varphi,t)}{\partial\varphi}\right\}. \tag{5}$$

The initial and boundary conditions on the probability density and the probability current for the potential profile (2) are as follows:

$$W(\varphi,0)=\delta(\varphi-\varphi_0),\quad W(+\infty,t)=0,\quad G(-\infty,t)=0.$$

The presence of fluctuations not only causes the finiteness of the life time of the S state [7], but also in a determined manner influences the turn-on delay time. Let us analyze such an influence.

3. The used method and the main result

The used method is based on the Laplace transformation of the FPE (5) and on the derivation of the time characteristics of the evolution of $W(\varphi,t)$ immediately from the solution of the equation for the probability-density Laplace transform

$$Y(\varphi,s)=\int_0^\infty W(\varphi,t)\mathrm{e}^{-st}\mathrm{d}t:$$

$$\frac{\mathrm{d}^2Y(\varphi,s)}{\mathrm{d}\varphi^2}+\frac{\mathrm{d}}{\mathrm{d}\varphi}\left[\frac{\mathrm{d}u(\varphi)}{\gamma\mathrm{d}\varphi}Y(\varphi,s)\right]-sBY(\varphi,s)=-B\delta(\varphi-\varphi_0),\tag{6}$$

where $B=1/(\gamma\omega_c)$. Note, that the Laplace transformed probability current is equal to

$$\hat{G}(\varphi,s)=\int_0^\infty G(\varphi,t)\mathrm{e}^{-st}\,\mathrm{d}t=-\frac{1}{B}\left[\frac{1}{\gamma}\frac{\mathrm{d}u(\varphi)}{\mathrm{d}\varphi}Y(\varphi,s)+\frac{\mathrm{d}Y(\varphi,s)}{\mathrm{d}\varphi}\right].\tag{7}$$

In accordance with Refs. [14–16], the characteristic time of the probability density evolution equals

$$\tau=\lim_{s\to0}\frac{s\hat{P}(s)-P(\infty)}{s[P(0)-P(\infty)]}=\lim_{s\to0}\frac{[P(0)-P(\infty)]-\left[\hat{G}(\varphi_2,s)-\hat{G}(\varphi_1,s)\right]}{s[P(0)-P(\infty)]},\tag{8}$$

where

$$P(t)=\int_{\varphi_1}^{\varphi_2}W(\varphi,t)\,\mathrm{d}\varphi,\quad \hat{P}(s)=\int_0^\infty P(t)\mathrm{e}^{-st}\mathrm{d}t=\int_{\varphi_1}^{\varphi_2}Y(\varphi,s)\,\mathrm{d}\varphi,\quad \varphi_1<\varphi_0<\varphi_2.$$

Note, that the time τ is defined as the time of the probability evolution $P(t)$ from $P(0)$ to $P(\infty)$ being equal to the mean escape time of the initial phase difference $\varphi=\varphi_0$ from the interval $[\varphi_1,\varphi_2]$ (Fig. 1). Thus, to obtain τ it is necessary to find the solution $Y(\varphi,s)$ of Eq. (6) for the potential (2) and evaluate the limit (8) for $s\to0$.

Expression (8) is exactly the limit with the aid of which the various time characteristics of the Brownian motion in arbitrary piecewise linear and piecewise parabolic potential profiles were obtained [14–16], and in particular this approach was used to find the turn-on delay time [12] in the parabolic approximation of the tilted potential profile (2). The point is that only for piecewise linear and piecewise parabolic potential profiles $u(\varphi)$ can one find the exact solution $Y(\varphi,s)$ of Eq. (6).

However, for an arbitrary $u(\varphi)$ and in particular for the potential profile (2) nobody knows this solution. That is why we will use a new approach to calculate the time τ. This approach is developed by Malakhov to obtain the time characteristics of stochastic transitions in nonlinear polystable dynamic systems under noise and will be published in detail for various types of potential profiles $u(\varphi)$. Here we sketch only the main features of this method, as applied to an arbitrary potential profile $u(\varphi)$ which tends fast enough to plus infinity for $x\to-\infty$ and to minus infinity for $x\to+\infty$.

Let us note in accordance with Eq. (8), that it is not necessary to know the whole function $\hat{G}(\varphi, s)$ and, consequently, the function $Y(\varphi, s)$, but their behavior at $s \to 0$ only.

For this reason let us introduce new functions $Z(\varphi, s)$, $H(\varphi, s)$ and expand them in a power series in s:

$$\begin{aligned} Z(\varphi, s) &\equiv sY(\varphi, s) = Z_0(\varphi) + sZ_1(\varphi) + s^2 Z_2(\varphi) + \dots, \\ H(\varphi, s) &\equiv s\hat{G}(\varphi, s) = H_0(\varphi) + sH_1(\varphi) + s^2 H_2(\varphi) + \dots \end{aligned} \tag{9}$$

In accordance with the limit theorems of the Laplace transform, one can get

$$Z_0(\varphi) = \lim_{s\to 0} sY(\varphi, s) = W(\varphi, \infty) = W_{st}(\varphi),$$

$$H_0(\varphi) = \lim_{s\to 0} s\hat{G}(\varphi, s) = G(\varphi, \infty) = G_{st}(\varphi).$$

It is obvious that for the discussed potential the steady-state (i.e. for $t \to \infty$) quantities of the probability density and the probability current equal zero, i.e. $Z_0(\varphi) \equiv 0$, $H_0(\varphi) \equiv 0$ for any finite values of φ.

Substituting Eq. (9) into Eq. (8) and taking into account that $P(0) = 1$, $P(\infty) = 0$, we get

$$\tau = \lim_{s\to 0} \left\{ \frac{1 - [H_1(\varphi_2) - H_1(\varphi_1)]}{s} - [H_2(\varphi_2) - H_2(\varphi_1)] - s[H_3(\varphi_2) - H_3(\varphi_1)] + \dots \right\}.$$

As will be demonstrated below, $H_1(\varphi_2) - H_1(\varphi_1) = 1$. Thus,

$$\tau = H_2(\varphi_1) - H_2(\varphi_2). \tag{10}$$

Substituting Eq. (9) into Eqs. (6) and (7) it is easy to find the differential equations for $H_k(\varphi)$, which have a simpler form than the original Eq. (6):

$$\begin{aligned} &\frac{dH_0(\varphi)}{d\varphi} = 0, \\ &\frac{dH_1(\varphi)}{d\varphi} = \delta(\varphi - \varphi_0), \\ &\frac{d^2 H_2(\varphi)}{d\varphi^2} + \frac{du(\varphi)}{d\varphi}\frac{dH_2(\varphi)}{d\varphi} = BH_1(\varphi). \end{aligned} \tag{11}$$

Using the boundary conditions $W(+\infty, t) = 0$ and $G(-\infty, t) = 0$, one can obtain from Eq. (11) $H_1(\varphi) = 1(\varphi - \varphi_0)$, which gives $H_1(\varphi_2) - H_1(\varphi_1) = 1$ and

$$H_2(\varphi) = \begin{cases} -B \int\limits_{\varphi_0}^{\varphi} e^{u(x)/\gamma} \int\limits_{-\infty}^{x} e^{-u(y)/\gamma} dy dx - B \int\limits_{\varphi}^{\infty} e^{u(x)/\gamma} dx \int\limits_{-\infty}^{\varphi} e^{-u(y)/\gamma} dy, & \varphi > \varphi_0, \\ -B \int\limits_{\varphi_0}^{\infty} e^{u(x)/\gamma} dx \int\limits_{-\infty}^{\varphi} e^{-u(y)/\gamma} dy, & \varphi < \varphi_0. \end{cases}$$

From this and Eq. (10) we have finally found the exact expression of the mean escape time of the random value φ from the interval $[\varphi_1, \varphi_2]$ for an arbitrary potential profile $u(\varphi)$ with $u(-\infty) = +\infty$, $u(+\infty) = -\infty$:

$$\tau = B \left\{ \int\limits_{\varphi_0}^{\varphi_2} e^{u(x)/\gamma} \int\limits_{\varphi_1}^{x} e^{-u(\varphi)/\gamma} d\varphi dx + \int\limits_{\varphi_1}^{\varphi_2} e^{-u(\varphi)/\gamma} d\varphi \int\limits_{\varphi_2}^{\infty} e^{u(\varphi)/\gamma} d\varphi \right\}. \tag{12}$$

If there is an absorbing boundary at the point $\varphi = \varphi_2$ (i.e. $u(\varphi) = -\infty$ for $\varphi \geq \varphi_2$) and $\varphi_1 = -\infty$, Eq. (12) transforms, as it should, to the well-known formula by Pontryagin of Ref. [17] for the mean first passage time,

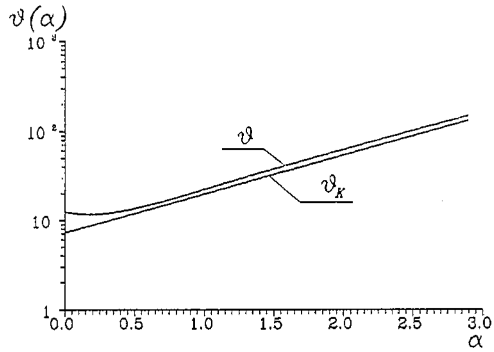

Fig. 2. The exact metastable state life time ϑ (13) and Kramers' time ϑ_K (14), vs. the potential barrier height α ($\vartheta = \omega_c\tau$, $\vartheta_K = \omega_c\tau_K$ $\varphi_2 = \pi$, $\varphi_0 = \arcsin(i)$, $\varphi_1 = -\pi$, $i = 0.5$).

$$\tau = B\int_{\varphi_0}^{\varphi_2} e^{u(x)/\gamma}\int_{-\infty}^{x} e^{-u(v)/\gamma}dv dx.$$

Thus, as follows from Eq. (12) the life time of the metastable state for the potential profile $u(\varphi) = 1-\cos\varphi - i\varphi$ has the form

$$\tau = B\left\{\int_{\varphi_0}^{\varphi_2} e^{-(\cos x+ix)/\gamma}\int_{\varphi_1}^{x} e^{(\cos\varphi+i\varphi)/\gamma}d\varphi dx + \int_{\varphi_1}^{\varphi_2} e^{(\cos\varphi+i\varphi)/\gamma}d\varphi\int_{\varphi_2}^{\infty} e^{-(\cos\varphi+i\varphi)/\gamma}d\varphi\right\}. \quad (13)$$

4. The analysis of the metastable-state life time

Let us analyze the formula (13) for different values of the current i and the fluctuation intensity γ. For $0 < i < 1$ and $\gamma \ll 1$ the asymptotic representation of the formula (13) may be also obtained by Kramers' method [18,8]:

$$\tau_K = \frac{2\pi}{\omega_c\sqrt{1-i^2}}e^{\alpha}, \quad (14)$$

where $\alpha = \Delta u/\gamma = (2\sqrt{1-i^2} + 2i(\arcsin i - \pi/2))/\gamma$ is the dimensionless potential barrier height (Fig. 1). The results of a computer simulation of Eq. (13) demonstrate that for $i > 0.3$ Eq. (14) holds true up to $\alpha \geq 1$ (see Fig. 2, where $\vartheta = \omega_c\tau$, $\vartheta_K = \omega_c\tau_K$, $\varphi_2 = \pi$, $\varphi_0 = \arcsin(i)$, $\varphi_1 = -\pi$, $i = 0.5$). For $i < 0.3$, however, the difference between τ (see Eq. (13)) and τ_K increases; the process of diffusion becomes slower because of the influence of neighbor potential barriers. For instance, at $i = 0.01$ the ratio $\tau/\tau_K \approx 40$ for $\alpha = 1$ and $\tau/\tau_K \approx 4$ for $\alpha = 10$. Actually, formula (13) is valid only for $i > 0$, because at $i = 0$ the process of diffusion from the initial well is going on very slowly and the definition (8) does not apply here. The results presented in the paper of Ref. [19] demonstrate that the time scale of diffusion from a potential well to a flat profile is proportional to $\sim e^{2\alpha}$, where α is the dimensionless depth of the well.

If the intensity of the thermal fluctuations is large, $\gamma \gg 1$, both for $i < 1$ and for $i > 1$ the influence of the cosinusoidal part of the potential becomes small and we can consider the potential $u(\varphi) \approx 1 - i\varphi$. Then τ approximately equals

$$\tau \approx \frac{1}{\omega_c}\left[\frac{\varphi_2-\varphi_0}{i} + \frac{\gamma}{i^2}\left\{1 - e^{i(\varphi_1-\varphi_0)/\gamma}\right\}\right], \quad (15)$$

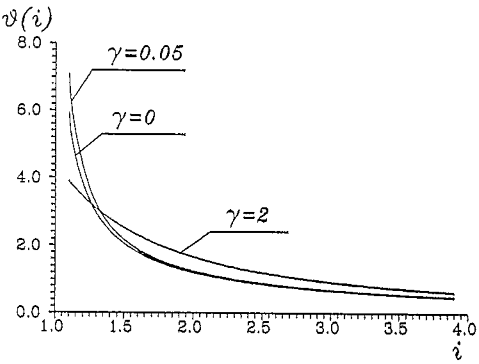

Fig. 3. The turn-on delay time ϑ, vs. the current i for different values of γ ($\vartheta = \omega_c\tau$, $\varphi_2 = \pi$, $\varphi_0 = \pi/2$, $\varphi_1 = -\pi$).

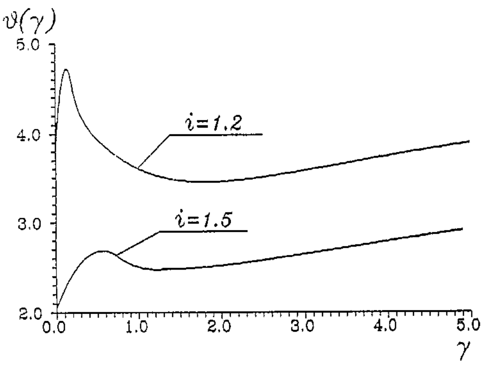

Fig. 4. The turn-on delay time ϑ, vs. the intensity of the thermal fluctuations γ for different values of i ($\vartheta = \omega_c\tau$, $\varphi_2 = \pi$, $\varphi_0 = \pi/2$, $\varphi_1 = -\pi$).

i.e. the life time increases proportionally to the increasing of the fluctuation intensity. However, at very large γ, $e^{i(\varphi_1-\varphi_0)/\gamma}$ tends to unity and we get the indeterminate form which may be evaluated as follows:

$$\tau \approx \frac{1}{\omega_c}\left[\frac{\varphi_2-\varphi_1}{i}\right], \tag{16}$$

so τ does not depend on the fluctuation intensity.

For $i > 1$ and $\gamma \ll 1$ one can obtain the following asymptotic representation of the formula (13):

$$\tau \approx \frac{1}{\omega_c}\left\{\frac{2}{\sqrt{i^2-1}}\arctan\left(\frac{i\tan(x/2)-1}{\sqrt{i^2-1}}\right)\Bigg|_{x=\varphi_0}^{x=\varphi_2} + \gamma\left[\frac{1}{2(i-\sin\varphi_2)^2}+\frac{1}{2(i-\sin\varphi_0)^2}\right] + \gamma^2\int_{\varphi_0}^{\varphi_2}\left[\frac{3\cos^2 x}{(i-\sin x)^5}-\frac{\sin x}{(i-\sin x)^4}\right]dx+\ldots\right\}. \tag{17}$$

If the fluctuation intensity is equal to zero ($\gamma = 0$), then formula (17) coincides with the formula obtained without any fluctuations in Ref. [6] and, in particular, with formula (4) for $\varphi_2 = \pi$ and $\varphi_0 = \pi/2$. Thus, the representation (17) shows that increasing of the fluctuation intensity for $\gamma \ll 1$ causes increasing of the

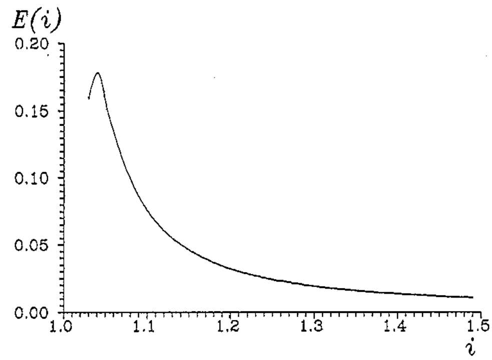

Fig. 5. The relative difference $E(i)$ between τ and τ_d, vs. the current i, demonstrating the effect of fluctuations on the turn-on delay time ($\gamma = 0.01$, $\varphi_2 = \pi$, $\varphi_0 = \pi/2$, $\varphi_1 = -\pi$).

metastable state life time (see Fig. 3, where $\vartheta = \omega_c \tau$; Figs. 3 and 4 were plotted for $\varphi_2 = \pi$, $\varphi_0 = \pi/2$, $\varphi_1 = -\pi$).

The effect of increasing of the turn-on delay may be considered in detail by plotting the dimensionless time ϑ as a function of the fluctuation intensity for different values of the current i. Fig. 4 demonstrates three different intervals of the $\vartheta(\gamma)$ curve behavior. It may be explained taking into account the competition of two factors: the variance increasing with increasing of the temperature and the influence of the returning force of the potential profile of Eq. (2).

When the phase variance is not so large, the main part of the phase probability distribution is located on the flat part near the point $\varphi = \pi/2$ and the influence of the fluctuations is stronger than that of the returning force. Further, when the magnitude of the variance becomes larger, the returning force quickly increases with the coordinate and we can see a decreasing of the turn-on delay time. Finally, when the variance value becomes large enough, we should take into account the influence of other potential periods and the asymptote of the $\vartheta(\gamma)$ curve may be represented by formulae (15) and (16). Note, in conclusion, that for $\gamma \ll 1$, as it should be, τ in both formula (14) and (17) does not depend on the value of φ_1 and coincides with the results of Ref. [20].

5. Conclusions

Application of JJ's as elements of logic devices supposes that the probability of thermally induced errors should be small enough, i.e. the life time of the "correct" state should be much larger than the operation cycle. Simple estimations following from the above presented formulae can demonstrate the boundary of the fluctuation intensity γ below which the logic element will function properly. Considering for the sake of simplicity the turn-on delay time as the main part of an operation cycle, one can see (Fig. 3) that in terms of the dimensionless time $\vartheta = \omega_c \tau$, the turn-on delay time is approximately equal to unity for $i \geq 2$ and $\vartheta \leq 7$ for $i \geq 1.1$. The prefactor of the formula (14) for $i = 0.8$ is of the order of 10; substituting $\alpha = 10$ into this formula, we get a value of S state life time equal to 10^5–10^6, which is well above the turn-on delay time. Taking into account that $\Delta u \approx 0.1$ for $i = 0.8$, it is enough to assume $\gamma \leq 0.01$ in order for α to be 10. The influence of thermal fluctuations on the turn-on delay time for $\gamma \leq 0.01$ is well predicted both by formula (17) and the corresponding formula in Ref. [12].

For example, it is interesting to plot the function $E(i) = (\tau/\tau_d - 1)$ for $\gamma = 0.01$, where τ is the exact turn-on delay time (17) and τ_d is described by formula (4) (see Fig. 5). The function $E(i)$ demonstrates the relative difference between τ and τ_d, and helps to choose the interval of the current i, in which the relative error $E(i)$ will be as small as necessary for concrete applications.

Let us note finally that the above obtained results are classical and do not take into account either shot or quantum fluctuations. The conditions for such an approach when the effect of the Coulomb blockade in junctions with a small capacitance will be masked by thermal fluctuations have been demonstrated in Refs. [21–23].

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