Suppression of Timing Errors in Short Overdamped Josephson Junctions

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The influence of fluctuations and periodical driving on temporal characteristics of short overdamped Josephson junctions is analyzed. We obtain the standard deviation of the switching time in the presence of a dichotomous driving force for arbitrary noise intensity and in the frequency range of practical interest. For sinusoidal driving the resonant activation effect has been observed. The mean switching time and its standard deviation have a minimum as a function of driving frequency. As a consequence the optimization of the system for fast operation will simultaneously lead to the minimization of timing errors.

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The rapid single flux quantum (RSFQ) electronic devices [1] are promising candidates to built a petaflop computer [2] due to the high operating frequencies of RSFQ elements close to 1 THz. Moreover, they are of particular interest in solid-state quantum information processing: the RSFQ circuitry may be used both for the realization of qubits and for the characterization of a macroscopic quantum behavior, e.g., readout electronics for quantum computing [3]. It is known that the processes going on in RSFQ devices are based on a reproduction of quantum pulses due to spasmodic changing by 2π of the phase difference of damped Josephson junctions (JJ). The major restriction in the development of RSFQ logic circuits is given by the influence of fluctuations [1,4]. Recently lots of investigations were performed to study pulse jitter and timing errors in RSFQ circuits [5,6]. Timing errors is one of reasons, limiting a 16-bit RSFQ microprocessor prototype [2] clock frequencies to 20 GHz instead of the theoretically predicted hundreds of gigahertz. Therefore, investigation of possible ways for suppression of pulse jitter of transmitting signals is a very important problem from a fundamental and a technological point of view and is also characteristic for different branches of physics, where a nonlinear element is driven either from an external source or from another element, as in neural networks.

For RSFQ circuits three different types of digital errors may be identified [5]. First, storage errors may occur during the passive storage of data: fluctuations can induce a 2π phase flip and thus switch the quantizing loop into the neighboring flux state. The analytical description of the mean time of such noise-induced flips, valid for arbitrary noise intensity, has been presented in [7]. Second, decision errors may be produced in the two junction comparator. This situation has been studied in detail in the linearized overdamped JJ model [8]. If an RSFQ circuit operates at high speed, another type of noise-induced error becomes important. Because of noise, the time interval between input and output pulses fluctuates. This type of error is called a “timing” error. Timing errors have also been studied in the linearized overdamped JJ model and with the assumption of instant change of external signals [5]. It is known that in a nonlinear system with a metastable state and noise the resonant activation (RA) effect [9–11] and noise enhanced stability (NES) [7,12] may be observed. These effects may play positive and negative role in the accumulation of fluctuational errors in RSFQ logic devices: the RA phenomenon minimizes timing errors, while the NES phenomenon increases the switching time. These effects, however, were not still observed in previous investigations due to the use of linearized models [5,8]. Moreover, the limiting frequencies of RSFQ devices and possible optimizations, in order to increase working frequencies and reduce timing errors in RSFQ circuits, is an open question.

This Letter is aimed at answering this question by investigating nonlinear noise properties of an overdamped JJ, subjected to periodic driving, to understand possible ways of RSFQ circuits optimization for high-frequency operation with minimal timing errors. To this end, we consider the dynamics of a short overdamped JJ, under a current I, given by the Langevin equation [4]:

$$\omega_c^{-1}\frac{d\varphi(t)}{dt} = -\frac{du(\varphi)}{d\varphi} - i_F(t), \qquad (1)$$

$$u(\varphi) = 1 - \cos\varphi - i(t)\varphi, \qquad i(t) = i_0 + f(t), \qquad (2)$$

where φ is the order parameter phase difference, $u(\varphi)$ is the dimensionless potential profile, $f(t)$ is the driving signal, $i = I/I_c$, with I_c the critical current, and $i_F(t) = I_F/I_c$, with I_F the random component of the current. Here $\omega_c = 2eR_NI_c/\hbar$ is the characteristic frequency of the JJ, R_N is the normal state resistance of a JJ, e is the electron charge, and $\hbar$ is the Planck constant. Let us consider only thermal fluctuations, then the random current may be represented by the white Gaussian noise: $\langle i_F(t)\rangle = 0$, $\langle i_F(t)i_F(t+\tau)\rangle = 2\gamma\delta(\tau)/\omega_c$. Here $\gamma = 2ekT/(\hbar I_c)$ is the dimensionless noise intensity, T is the temperature, and k is the Boltzmann constant. Let, initially, the JJ be biased by a current smaller than the critical one ($i_0 < 1$), and the junction is in the superconductive state. The current pulse $f(t)$, such that $i(t) = [i_0 + f(t)] > 1$, switches the junction into the resistive state. However, an output pulse will be born not immediately, but at the later time. Such a time is the switching time [4], and, due to fluctuations, is a random quantity and may be characterized by its mean value and standard deviation. As an example of a driving with sharp fronts, we consider the dichotomous signal $f(t) = A\,\mathrm{sgn}[\sin(\omega t)]$, and as an example of a driving with smooth fronts—sinusoidal signal $f(t) = A\,\sin(\omega t)$. In spite that we consider the periodic driving, which makes the results more evident, it is obvious that below the cutoff frequency the switching occurs during half of the first period, so other periods are not important. Besides, our adiabatic analysis may easily be generalized to an arbitrary form of $f(t)$. In computer simulations we set $\omega_c = 1$ and, therefore, in plots ω is normalized to ω_c.

Let us investigate the mean switching time (MST) and its standard deviation (SD). These quantities may be introduced as characteristic time scales of the evolution of the probability $P(t) = \int_{\varphi_1}^{\varphi_2} W(\varphi, t)d\varphi$ to find the phase within one period of the potential profile (2). Therefore we choose $\varphi_2 = \pi$, $\varphi_1 = -\pi$, and the initial distribution at the bottom of a potential well, i.e., $\varphi_0 = \arcsin(i_0)$. The mean switching time and the standard deviation are

$$\tau = \langle t \rangle = \int_0^\infty t w(t)dt, \qquad \sigma = \sqrt{\langle t^2 \rangle - \langle t \rangle^2},$$
$$w(t) = \frac{\partial P(\varphi_0, t)}{\partial t[P(\varphi_0, \infty) - P(\varphi_0, 0)]}. \tag{3}$$

Let us first consider the case of dichotomous driving, $f(t) = A\,\mathrm{sgn}[\sin(\omega t)]$. From the Langevin Eq. (1), we calculate the MST and its SD. In Fig. 1 we report the behaviors of MST $\tau(\omega)$ and SD $\sigma(\omega)$ versus driving frequency for two values of noise intensity: $\gamma = 0.2$, 0.02. For dichotomous driving both MST and its SD do not depend on the driving frequency below a certain cutoff frequency, above which the characteristics degrade. In the frequency range 0–0.2ω_c, therefore, we can describe the effect of dichotomous driving by time characteristics in a constant potential. The exact analytical expression of MST and its asymptotic expansion are, for arbitrary γ, [7]

$$\tau_c(\varphi_0) = \frac{1}{\gamma\omega_c}\left\{\int_{\varphi_0}^{\varphi_2} e^{u(x)/\gamma}\int_{\varphi_1}^{x} e^{-u(\varphi)/\gamma}d\varphi dx + \int_{\varphi_1}^{\varphi_2} e^{-u(\varphi)/\gamma}d\varphi\int_{\varphi_2}^{\infty} e^{u(\varphi)/\gamma}d\varphi\right\}, \tag{4}$$

and for $\gamma \ll 1$

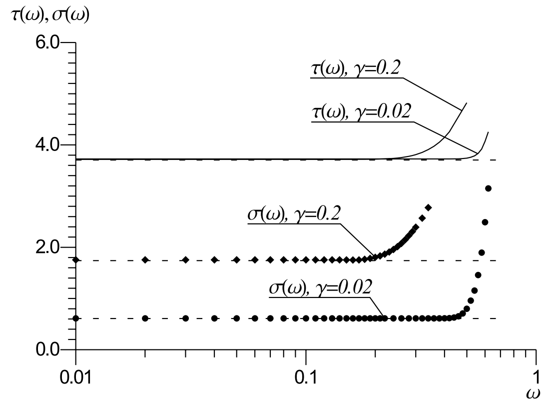

FIG. 1. The MST $\tau(\omega)$ and SD $\sigma(\omega)$ as functions of frequency for dichotomous driving, for two values of noise intensity: $\gamma = 0.2, 0.02$, and $i_0 = 0.5$, $i = 1.5$. The results of computer simulations are $\tau(\omega)$ (solid line) and $\sigma(\omega)$ (diamonds and circles). Dashed lines are theoretical results (5) and (8).

$$\tau_c(\varphi_0) = \frac{f_1(\varphi_2) - f_1(\varphi_0) + \gamma[f_2(\varphi_2) + f_2(\varphi_0)] + \cdots}{\omega_c}, \tag{5}$$

$$f_1(x) = \frac{2}{\sqrt{i^2 - 1}}\arctan\left(\frac{i\tan(x/2) - 1}{\sqrt{i^2 - 1}}\right), \tag{6}$$
$$f_2(x) = 1/[2(i - \sin x)^2].$$

Following Ref. [11], the exact expression for $\tau_{2c} = \langle t^2 \rangle$ in a time-constant potential may be derived as

$$\tau_{2c}(\varphi_0) = \tau_c^2(\varphi_0) - \frac{2}{(\gamma\omega_c)^2}\int_{\varphi_0}^{\varphi_2} e^{-u(x)/\gamma}H(x)dx - \frac{2}{(\gamma\omega_c)^2}H(\varphi_0)\int_{\varphi_1}^{\varphi_0} e^{-u(x)/\gamma}dx, \tag{7}$$

where $H(x) = \int_x^\infty e^{u(v)/\gamma}\int_v^{\varphi_2} e^{-u(y)/\gamma}\int_y^\infty e^{u(z)/\gamma}dzdydv$, and $\tau_c(\varphi_0)$ is given by (4). The asymptotic expression of $\sigma = \sqrt{\tau_{2c} - \tau_c^2}$ in the small noise limit $\gamma \ll 1$ is

$$\sigma(\varphi_0) = \frac{1}{\omega_c}\sqrt{2\gamma[F(\varphi_0) + f_3(\varphi_0)] + \cdots}, \tag{8}$$

$$F(\varphi_0) = f_1(\varphi_2)f_2(\varphi_2) - 2f_1(\varphi_0)f_2(\varphi_0) + f_1(\varphi_0)f_2(\varphi_0) + \frac{f_1(\varphi_2) - f_1(\varphi_0)}{[i - \sin(\varphi_0)]^2},$$

$$f_3(\varphi_0) = \int_{\varphi_0}^{\varphi_2}\left[\frac{\cos(x)f_1(x)}{[\sin(x) - i]^3} - \frac{3}{2[\sin(x) - i]^3}\right]dx.$$

The comparison between the asymptotic theoretical results [Eqs. (5) and (8)] and simulations is reported in

Fig. 1. The agreement is very good within the frequency range 0–0.2ω_c. It is interesting to see that the SD of the switching time scales as the square root of noise intensity. The dependencies of the MST and its SD on the bias current are presented in Fig. 2 for $\gamma = 0.001$. The agreement between theoretical results and simulations is very good for all the bias current values investigated. In the low noise limit, Eq. (5) actually gives the same results as presented in [5], since the largest contribution to the MST comes from the deterministic term $\tau_c(\varphi_0) = \frac{1}{\omega_c}\{f_1(\varphi_2) - f_1(\varphi_0)\}$. However, formula (8), in some cases, significantly deviates from the results of linearized calculations [5]. For the case $\gamma = 0.001$, the authors of [5] have got $\sigma = 0.4\omega_c^{-1}$ for $i = 1.2$, while we get $\sigma = 0.436\omega_c^{-1}$, but for larger current $i = 1.5$ the discrepancy is larger: $\sigma = 0.06\omega_c^{-1}$ in [5], and we get $\sigma = 0.14\omega_c^{-1}$. In the inset of Fig. 2 we report the behavior of SD, Eq. (8), as a function of noise intensity γ, for $i = 1.2$ and $i = 1.5$. The agreement with computer simulations is very good for noise intensity values up to $\gamma = 0.05$. Therefore not only low temperature ($\gamma \leq 0.001$) but also high temperature devices may be described by Eqs. (5) and (8). If noise intensity is rather large, the phenomenon of NES may be observed: the MST increases with the noise intensity, as it may be easily seen from Eq. (5). In the design of large arrays of RSFQ elements, operating at high frequencies, it is very important to consider this effect; otherwise it may lead to malfunctions due to the errors accumulation.

Now let us consider the case of sinusoidal driving $f(t) = A\sin(\omega t)$. The corresponding time characteristics may be derived using the modified adiabatic approximation [10,11]

$$P(\varphi_0, t) = \exp\left\{-\int_0^t \frac{1}{\tau_c(\varphi_0, t')} dt'\right\}, \qquad (9)$$

where $\tau_c(\varphi_0, t')$ is given by (4). While for the case of dichotomous driving the value of initial current i_0 has a weak effect on temporal characteristics in agreement with [5], for the case of sinusoidal driving the value of i_0 is more important, since it also defines the potential barrier height. We focus now on the current value $i = 1.5$, because $i = 1.2$ is too small for high-frequency RSFQ applications. In Fig. 3 the MST as a function of the driving frequency for different values of bias current $i_0 = 0.5, 0.8$ is shown. For smaller i_0 the switching time is larger, since $\varphi_0 = \arcsin(i_0)$ depends on i_0. On the other hand, the bias current i_0 must not be too large, since it will lead, in absence of driving, to the reduction of the mean lifetime of superconductive state (4), i.e., to the increase of storage errors. Therefore, there must be an optimal value of bias current i_0, giving minimal switching time and acceptably small storage errors. Following [5], storage errors are acceptably small up to $i_0 = 0.99$ for $\gamma = 0.001$. In the inset of Fig. 3 the MST as a function of driving frequency is presented for the following parameters: $i_0 = 0.5$, $A = 1$, $\gamma = 0.02, 0.05, 0.5$. We observe the phenomenon of resonant activation: MST has a minimum as a function of driving frequency. The approximation (9) does not describe the resonant activation effect at high frequencies, but it works rather well below $0.1\omega_c$, which is enough for practical applications. It is interesting to see that near the minimum the MST has a very weak dependence on the noise intensity; i.e., in this range of the signal frequency the noise is effectively suppressed. We observe also the NES phenomenon. There is a frequency range, around 0.2–0.4ω_c for $i_0 = 0.5$ and around 0.3–0.5ω_c for $i_0 = 0.8$, where the switching time increases with an increase of noise intensity. The NES effect increases for smaller i_0 because the potential barrier disappears for a short time interval within the driving period [12] and the potential is more flat, so noise has more chances to prevent

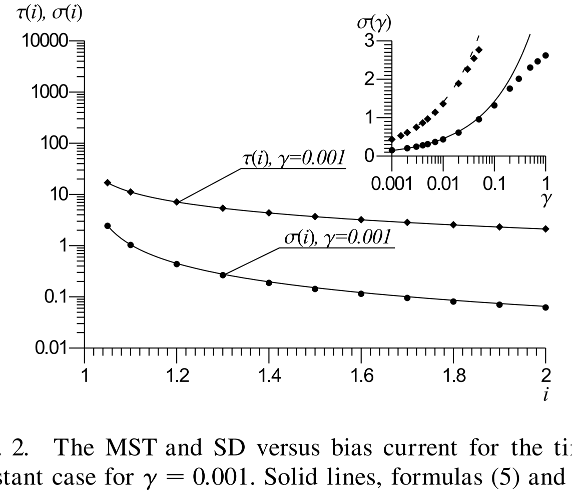

FIG. 2. The MST and SD versus bias current for the time-constant case for $\gamma = 0.001$. Solid lines, formulas (5) and (8); diamonds and circles, results of computer simulation. Inset: the SD versus noise intensity for the time-constant case ($i > 1$). Solid line, formula (8); circles, results of computer simulation for $i = 1.5$; dashed line, formula (8); diamonds, results of computer simulation for $i = 1.2$.

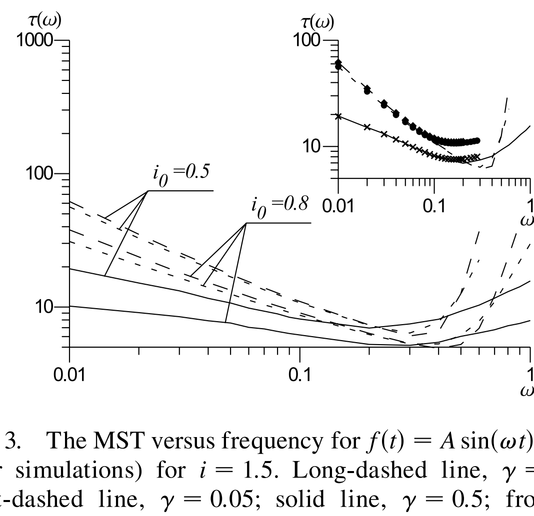

FIG. 3. The MST versus frequency for $f(t) = A\sin(\omega t)$ (computer simulations) for $i = 1.5$. Long-dashed line, $\gamma = 0.02$; short-dashed line, $\gamma = 0.05$; solid line, $\gamma = 0.5$; from top to bottom, $i_0 = 0.5$ and $A = 1$, $i_0 = 0.8$ and $A = 0.7$. Inset: simulations versus theoretical results obtained from Eq. (9) for $i_0 = 0.5$, $A = 1$, and $\gamma = 0.02$ (diamonds), $\gamma = 0.05$ (circles), $\gamma = 0.5$ (crosses).

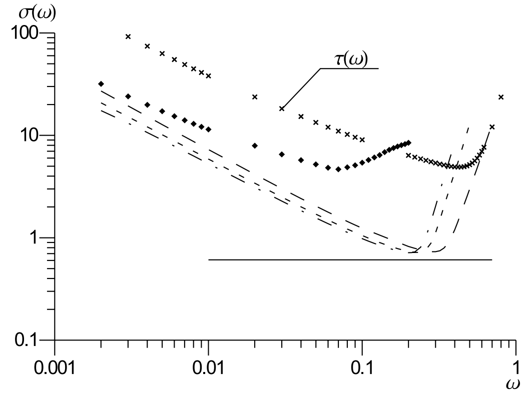

FIG. 4. The SD versus frequency for $f(t) = A\sin(\omega t)$ and $\gamma = 0.02$. Computer simulations: dash-dotted line, $i_0 = 0.3$, $A = 1.2$; short-dashed line, $i_0 = 0.5$, $A = 1$; long-dashed line, $i_0 = 0.8$, $A = 0.7$. The MST is given by crosses for comparison ($i_0 = 0.8$, $A = 0.7$). Formula (8), solid line; formula (9), diamonds.

the phase to move down and delay switching process. It is clear that this effect may be avoided, if the operating frequency does not exceed $0.2\omega_c$. Besides, the SD also increases above $0.2\omega_c$ (Fig. 4).

The plots of SD versus driving frequency for $\gamma = 0.02$, $i = 1.5$, and different values of $i_0 = 0.3, 0.5, 0.8$ are shown in Fig. 4. The approximation (9) is not as good for SD as it is for MST, even if the qualitative behavior of SD is recovered. We see that the minimum of $\sigma(\omega)$, for $\gamma = 0.02$, is located near the corresponding minimum of $\tau(\omega)$ (Fig. 3). For the SD the optimal frequency range where the noise-induced error will be minimal is from 0.1 to 0.3 for the considered range of parameters. It is interesting to see that, near the minimum, the SD for sinusoidal driving actually coincides with SD for dichotomous driving (8). This means that, even in the case of smooth driving, the limiting value of SD may nearly be reached, but the RSFQ circuit must be properly optimized.

Finally, we note that the close location of minima of MST and its SD means that the optimization of RSFQ circuit for fast operation will simultaneously lead to the minimization of timing errors in the circuit, which is the main result of this Letter.

In the present Letter, we reported an analytical and numerical analysis of influence of fluctuations and periodic driving on temporal characteristics of the JJ. For the case of dichotomous driving the analytical expression of standard deviation of switching time works in a practically interesting frequency range and for arbitrary noise intensity. For the case of sinusoidal driving the resonant activation effect has been observed in the considered system: mean switching time has a minimum as a function of driving frequency. Near this minimum the standard deviation of switching time takes also a minimum value. The RA phenomenon was observed very recently in the underdamped JJ [13]. Our theoretical investigation could motivate experimental work in overdamped JJs as well. Utilization of this effect in fact allows one to suppress time jitter in practical RSFQ devices and, therefore, allows one to significantly increase working frequencies of RSFQ circuits. Our study is not only important to understand the physics of fluctuations in a JJ to improve the performance of complex digital systems, but also in nonequilibrium statistical mechanics of dissipative systems, where noise assisted switching between metastable states takes place [9–13] and in neural networks.

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