

## Quantum corrections to the conductivity in two-dimensional systems: Agreement between theory and experiment

G. M. Minkov, O. E. Rut, A. V. Germanenko, and A. A. Sherstobitov

*Institute of Physics and Applied Mathematics, Ural State University, 620083 Ekaterinburg, Russia*

V. I. Shashkin, O. I. Khrykin, and V. M. Daniltsev

*Institute of Physics of Microstructures of RSA, 603600 Nizhni Novgorod, Russia*

(Received 10 May 2001; revised manuscript received 14 August 2001; published 28 November 2001)

Quantum corrections to the conductivity have been studied in the two types of low-mobility two-dimensional heterostructures: those with doped quantum well, and doped barriers. The consistent analysis shows that for the structures where electrons occupy the states only in the quantum well, all the temperature and magnetic field dependences of the components of resistivity tensor are well described by the theories of the quantum corrections. Contribution of the electron-electron interaction to the conductivity has been reliably determined for the structures with different electron density. A possible reason of large scatter in experimental data relating to the contribution of electron-electron interaction, obtained in previous papers, is analyzed. The role of the carriers occupying the states of the doped layers is discussed.

DOI: 10.1103/PhysRevB.64.235327

PACS number(s): 73.20.Fz, 73.61.Ey

### I. INTRODUCTION

The quantum corrections to the Drude conductivity in disordered metals and doped semiconductors have been intensively studied since 1980.<sup>1,2</sup> Two mechanisms lead to these corrections: (i) the interference of the electron waves propagating in opposite directions along closed paths and (ii) the electron-electron interaction (EEI). These corrections increase with decreasing temperature and/or increasing disorder and largely determine the low temperature transport in two-dimensional (2D) systems. Recently, the different behavior of the conductivity with decrease of temperature  $T$  has come to light: (i) the conductivity decreases monotonically for one type of the systems and (ii) the conductivity decreases at sufficiently high temperature, but reveals surprising growth at low enough temperature for other ones.<sup>3</sup>

It is commonly accepted that decrease of the conductivity for the first type of the systems results from temperature dependence of the quantum corrections that are negative and logarithmically diverge at  $T \rightarrow 0$ . In such systems the crossover from the weak localization (WL) regime where the corrections are small compared with the Drude conductivity to the strong localization (SL) regime is observed. The role of interference and electron-electron interaction in this crossover is of special interest and attracts much attention in recent years.

As for the second type of the 2D systems, no general consensus on the origin of metallic behavior has been reached as yet. The study of the role of the EEI and interference can be useful for understanding the origin of the metalliclike behavior of conductivity in such systems.

The WL-SL crossover was intensively studied in thin metal films. It is generally assumed that the EEI has a crucial role because the interference is suppressed by the strong spin-orbit interaction in metals. Different aspects of the WL-SL crossover were studied in semiconductor 2D structures but there is no conventional view on the role of EEI and interference in this crossover up to now. Moreover, the

magnitudes of the EEI and interference corrections to the conductivity have not been well established experimentally in the WL regime, when the theories of the quantum corrections are applicable. It especially concerns the EEI contribution. It was shown in Refs. 1,4 that the EEI contributes to  $\sigma_{xx}$  only. For  $g\mu_B B/kT \lesssim 1$  the correction has the form

$$\Delta\sigma_{xx}^{ee} = G_0 \left( 1 + \frac{3}{4}\lambda \right) \ln \left( \frac{kT\tau}{\hbar} \right), \quad (1)$$

whereas for  $g\mu_B B/kT \gg 1$  it is given by

$$\Delta\sigma_{xx}^{ee} = G_0 \left( 1 + \frac{1}{4}\lambda \right) \ln \left( \frac{kT\tau}{\hbar} \right). \quad (2)$$

Here,  $G_0 = e^2/(2\pi^2\hbar)$ ,  $\tau$  is the momentum relaxation time,  $\lambda$  is a function of  $k_F/K^5$  with  $k_F$  as the Fermi quasimomentum and  $K$  as the screening parameter, which for 2D case is equal to  $2/a_B$ , where  $a_B$  is the effective Bohr radius. For the most intensively studied Si MOS structures, the transition to the case  $g\mu_B B/kT \gg 1$  occurs within the actual range of temperature and magnetic field. It results in complicated magnetic field dependence of the resistance: decreasing at low magnetic field it increases at high ones. This fact makes the quantitative interpretation of experimental data difficult.

For the electron 2D systems based on GaAs, in which the electron  $g$  factor is much smaller than that in Si, the condition  $g\mu_B B/kT \lesssim 1$  is fulfilled in wide range of temperature and magnetic field except extremely low temperature or very high magnetic field. Therefore, the experimental results can be interpreted in the most simple way for these systems. The multiplier before logarithm in Eq. (1) is determined experimentally and just its value is shown in Fig. 1 as function of  $k_F/K$ . Theoretical curve from Ref. 5 is also shown in the figure. The large scatter in the experimental data shows that there are no reliable data on the contribution of the EEI and it is impossible to conclude whether the theory describes the experiment.

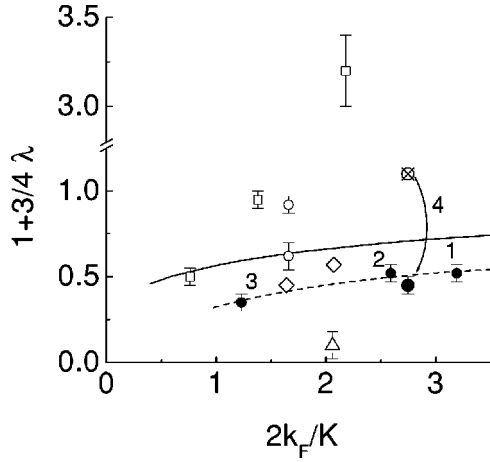


FIG. 1. The value of multiplier  $(1 + 3/4\lambda)$  in Eq. (2) as function of  $k_F/K$ . Symbols are the experimental results from Refs. 8 ( $\square$ ), 9 ( $\circ$ ), 10 ( $\triangle$ ), 11 ( $\diamond$ ), and our data ( $\bullet$ ,  $\otimes$ ). The solid curve represents the theoretical result from Ref. 5, dotted line is the guide for an eye. Arrow indicates the shift of experimental point for structure 4 after extraction of temperature dependence of electron mobility as described in Sec. VII.

From our point of view, before the discussion of the very interesting problem concerning the role of the EEI in WL-SL crossover and in the new low temperature metallic phase, it would be essential to acquire reliable data in the WL regime far from the WL-SL crossover. This is the problem our paper is devoted to.

Let us discuss what type of semiconductor heterostructures would be most suitable for the quantitative study of quantum corrections to the conductivity in 2D systems. First of all, the Drude conductivity  $\sigma_0$  should be high so that  $\sigma_0/(\pi G_0) = k_F l \gg 1$ , where  $l$  is the mean-free path. Namely, in this case the WL theory can be applied. On the other hand, the quantum corrections must not be very small, lest the measurement accuracy restrict the quantitative analysis. This means that the electron scattering must be strong enough, i.e., the mobility must not be high. The investigations should be carried out on the single-quantum-well heterostructure with one subband occupied, as the theories of quantum corrections have been developed mainly for such a case. Quantum well should be symmetric in shape. It allows us to eliminate the peculiarities caused by the spin-orbit interaction<sup>6,7</sup> and to neglect the spin effects under analysis of experimental data. It should be the structure with electrons located in the quantum well only, i.e., with an empty doped (modulation or  $\delta$ ) layer. It enables one to avoid the shunting of the quantum well. Moreover, if there are carriers in the doped layer, their redistribution within the layer with temperature change can lead to the temperature-dependent disorder, and, hence, to an additional temperature dependence of the mobility.<sup>12</sup>

Thus, two types of structures meet the following requirements: (i) the structures with doped quantum well and (ii) the structures with symmetrically doped barriers and low carrier density, when the Fermi level lies significantly lower than any states in doped layers. Exactly these types of structures will be reported in this work.

## II. THEORETICAL BASIS

In this section we present the main theoretical results which will be used in the analysis of the experimental data.<sup>1</sup> Note that the theories referred are valid when  $k_F l \gg 1$  and the quantum corrections to the conductivity are small compared with the Drude conductivity.

Without an external magnetic field the total quantum correction to conductivity is

$$\delta\sigma(T) = G_0 \left[ \ln\left(\frac{\tau}{\tau_\phi(T)}\right) + \left(1 + \frac{3}{4}\lambda\right) \ln\left(\frac{kT\tau}{\hbar}\right) \right], \quad (3)$$

where  $\tau_\phi$  is the phase-breaking time. The first term in Eq. (3) is the interference correction, the second one is the EEI contribution. At low temperatures the phase-breaking time is determined by inelasticity of the electron-electron interaction and is

$$\tau_\phi^{-1} = \frac{kT}{\hbar} \frac{2\pi G_0}{\sigma_0} \ln\left(\frac{\sigma_0}{2\pi G_0}\right). \quad (4)$$

The value of  $\lambda$  was obtained in Ref. 5

$$\lambda = 4 \left[ 1 - 2 \frac{(1 + \frac{1}{2}F) \ln(1 + \frac{1}{2}F)}{F} \right], \quad (5)$$

where

$$F = \int \frac{d\theta}{2\pi} \left[ 1 + \frac{2k_F}{K} \sin\frac{\theta}{2} \right]^{-1}. \quad (6)$$

In a magnetic field the classical conductivity tensor has the following form:

$$\sigma_{xx}^0 = \frac{en\mu}{1 + \mu^2 B^2}, \quad (7)$$

$$\sigma_{xy}^0 = \frac{en\mu^2 B}{1 + \mu^2 B^2}. \quad (8)$$

The electron-electron interaction contributes to  $\sigma_{xx}$  only [see Eqs. (1) and (2) for  $\Delta\sigma_{xx}^{ee}$ ], whereas  $\Delta\sigma_{xy}^{ee} = 0$ . It is easy to show that the magnetoresistance

$$\rho_{xx}(B, T) = \frac{\sigma_{xx}^0(B) + \Delta\sigma_{xx}^{ee}(T)}{[\sigma_{xy}^0(B)]^2 + [\sigma_{xx}^0(B) + \Delta\sigma_{xx}^{ee}(T)]^2} \quad (9)$$

is parabolic in the form when  $\Delta\sigma_{xx}^{ee} \ll \sigma_{xx}^0$ :

$$\rho_{xx}(B, T) \approx \frac{1}{\sigma_0} - \frac{1}{\sigma_0^2} (1 - \mu^2 B^2) \Delta\sigma_{xx}^{ee}(T). \quad (10)$$

So,  $\rho_{xx}$ -versus- $B$  curves for different temperatures should cross one another at fixed point  $B_{cr} = 1/\mu$  and the value of  $\rho_{xx}^{-1}(B_{cr})$  should be equal to the Drude conductivity.<sup>13</sup>

The interference correction to the conductivity gives the contributions to both  $\sigma_{xx}$  and  $\sigma_{xy}$  but their ratio is such that  $\rho_{xy}$  remains unchanged. Within the framework of the diffusion approximation which is valid when  $\tau_\phi/\tau \gg 1$  and  $B < B_{tr} = \hbar/(2el^2)$  the magnetic field dependence of  $\Delta(1/\rho_{xx}^{in}) = 1/\rho_{xx}(B) - 1/\rho(0)$  is described by the well-known expression<sup>6</sup>

TABLE I. Sample parameters.

Structure	1	2	3	4
$\sigma_0(10^{-4} \text{ Ohm}^{-1})$	$4.13 \pm 0.02$	$3.55 \pm 0.03$	$1.90 \pm 0.05$	$6.50 \pm 0.05$
$n(10^{12} \text{ cm}^{-2})$	$1.35 \pm 0.05$	$0.87 \pm 0.02$	$0.19 \pm 0.02$	$1.0 \pm 0.05$
$\tau(10^{-14} \text{ sec})$	6.5	8.8	21	13.7
$B_{tr}(\text{T})$	0.25	0.21	0.16	0.076
$k_F l$	10.7	9.2	4.9	16.6
$B_{cr}(\text{T})$	5.22	4.15	1.66	4.14
$\rho_{xx}^{-1}(B_{cr})(10^{-4} \text{ Ohm}^{-1})$	3.95	3.38	1.67	6.45
$\mu^{-1}(\text{T})^a$	5.26	4.17	1.64	2.50
$p$	$0.95 \pm 0.1$	$0.8 \pm 0.2$	$0.9 \pm 0.1$	$0.9 \pm 0.2$
$1 + 3/4\lambda$	$0.52 \pm 0.05$	$0.52 \pm 0.05$	$0.35 \pm 0.05$	$0.45 \pm 0.05$

<sup>a</sup>The value of mobility has been determined as  $\mu = \rho_{xy}/(\rho_{xx}B)$  at  $B = B_{cr}$ .

$$\Delta[1/\rho_{xx}^{\text{int}}(B)] = \alpha G_0 \left\{ \psi\left(\frac{1}{2} + \frac{\tau B_{tr}}{\tau_\varphi B}\right) - \psi\left(\frac{1}{2} + \frac{B_{tr}}{B}\right) - \ln\left(\frac{\tau}{\tau_\varphi}\right) \right\}, \quad (11)$$

where  $\psi(x)$  is a digamma function, the value of  $\alpha$  is equal to unity. The magnetic field dependence of the interference correction beyond the diffusion approximation was studied in Refs. 14–19. The analytical expression suitable for the fit of the experimental data was not obtained. However, as shown in Ref. 19 the use of Eq. (11) for the fit of  $\Delta(1/\rho_{xx}^{\text{int}})$ -vs- $B$  curve in this regime gives the value of  $\tau_\varphi$  very close to true one and the prefactor  $\alpha$  less than unity.

It follows from Eqs. (10) and (11) that the interference correction gives the strong magnetic field dependence of the resistivity at  $B \leq B_{tr}$ , whereas the EEI does it at magnetic field  $B \geq B_{cr} = 1/\mu$ . Since the ratio  $B_{cr}/B_{tr}$  is equal to  $2k_F l$ , these magnetic field ranges are well separated. Thus, an external magnetic field allows to separate the interference and interaction contributions to the conductivity. To answer the question ‘‘Does the theory of quantum corrections agree with experiment in 2D systems?,’’ all the theoretical predictions given above should be checked step by step.

### III. SAMPLES

The heterostructures with 50 Å  $\text{In}_{0.15}\text{Ga}_{0.85}\text{As}$  single quantum well in GaAs with Si  $\delta$ -doping layers were investigated. Two types of heterostructures were studied: with doped quantum well (structures 1 and 2), and with doped barrier (structures 3 and 4). In the first case the  $\delta$  layer was arranged in the center of quantum well. In the second one, two  $\delta$  layers, separated by the 60 Å GaAs spacer, were disposed on each side of the quantum well. The thickness of undoped GaAs cap layer was 3000 Å for all the structures. The samples were mesa etched into standard Hall bridges. The parameters of the structures are presented in Table I.

### IV. TEMPERATURE DEPENDENCE OF CONDUCTIVITY AT HIGH MAGNETIC FIELD. CONTRIBUTION OF ELECTRON-ELECTRON INTERACTION

The experimental magnetic field dependences of  $\rho_{xx}$  and  $\rho_{xy}$  measured for structure 2 at different temperatures are

presented in Figs. 2(a),2(b). The two different magnetic field ranges are evident: the range of sharp dependence of  $\rho_{xx}$  at low field  $B \leq 0.5-1$  T, and the range of moderate dependence which is close to parabolic one at higher field. All  $\rho_{xx}$ -vs- $B$  curves cross each other at fixed magnetic field  $B_{cr} = 4.15$  T. This value is close to  $1/\mu$  (see Table I). The Hall resistance is practically linear with magnetic field. However, despite the strong degeneracy of electron gas [ $E_F/(kT) > 100$ , where  $E_F$  is the Fermi energy] the Hall resistance decreases with increasing temperature. Low-magnetic-field behavior of  $\rho_{xx}$ , which is a consequence of suppression of the interference correction by magnetic field, will be discussed in the next section.

At high magnetic field ( $B > 1-2$  T),  $\rho_{xx}(B, T)$  and  $\rho_{xy}(B, T)$  differ from the classical behavior following from Eqs. (7) and (8) by contribution of electron-electron interaction only. To assure in this fact we plot the temperature dependences of the conductivity tensor components in Fig. 3. It is clearly seen that the change of  $\sigma_{xx}$  with temperature does not depend on  $B$  and significantly larger than that of  $\sigma_{xy}$ . Namely, such a behavior is in full agreement with the predictions of the EEI theory. Absence of the temperature dependence of nondiagonal component of conductivity tensor  $\sigma_{xy}$  allows us to attribute the temperature dependence of the

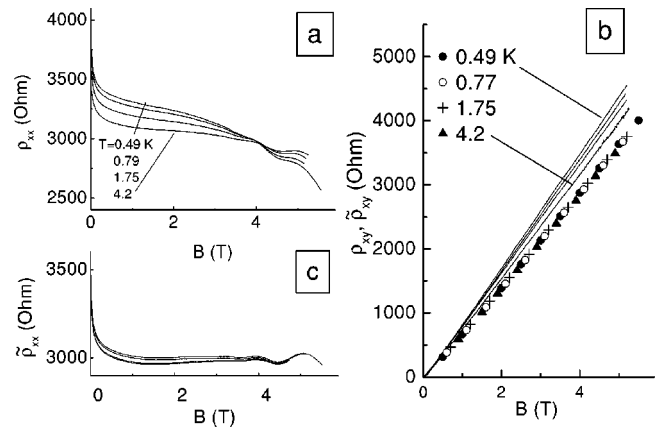


FIG. 2. Magnetic field dependences of  $\rho_{xx}$  (a),  $\rho_{xy}$  (lines) and  $\tilde{\rho}_{xy}$  (symbols) (b), and  $\tilde{\rho}_{xx}$  (c) for different temperatures for structure 2.

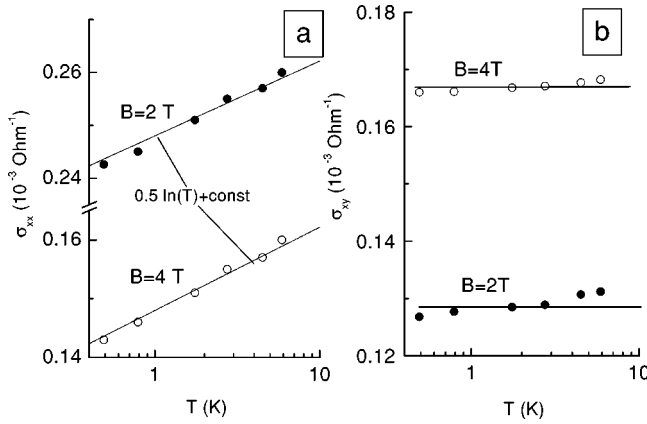


FIG. 3. Temperature dependence of  $\sigma_{xx}$  (a) and  $\sigma_{xy}$  (b) for two magnetic fields for structure 2.

diagonal component  $\sigma_{xx}$  to contribution of the EEI. So, we are able to determine the multiplier before logarithm in Eq. (1) from the slope of  $\sigma_{xx}$ -vs- $\ln T$  dependence [see Fig. 3(a)]. The result is  $(1 + 3/4\lambda) = 0.5 \pm 0.1$ .

Notice that practically in all the papers where the EEI was studied,  $\sigma_{xy}$  temperature independence was not demonstrated over the magnetic field range where the EEI contribution was determined. Below we will show that the existence of  $\sigma_{xy}$  temperature dependence introduces a large error into the determination of  $(1 + 3/4\lambda)$ .

Now when we have determined the EEI contribution, let us extract it from  $\sigma_{xx}$ , invert the conductivity tensor and obtain the components  $\tilde{\rho}_{xx}$  and  $\tilde{\rho}_{xy}$  without this correction. The result is presented in Figs. 2(b),2(c). Disappearance of the temperature dependence of  $\tilde{\rho}_{xy}$  and  $\tilde{\rho}_{xx}$  confirms the correctness of determination of the EEI contribution to the conductivity, and an absence of any mechanisms that can lead to an additional temperature dependence of the conductivity. We would like to mention that after extraction of the EEI contribution the values of electron density determined by different ways: (i)  $n = B/(e\tilde{\rho}_{xy})$ , (ii)  $n = B_{cr}/[e\rho_{xx}(B_{cr})]$ , and (iii) from the Shubnikov–de Haas oscillations, are very close to each other and lie within the error interval given in Table I.

## V. LOW-FIELD MAGNETORESISTANCE. INTERFERENCE CORRECTION TO THE CONDUCTIVITY

Let us consider the low magnetic field range. The temperature dependence of  $\rho_{xx}$  in this range is determined by both the EEI and interference contributions whereas the magnetic field dependence is determined by interference contribution only, because  $\rho_{xx}(B)$  is unaffected by the EEI at  $B \ll 1/\mu$ . Thus, the dependence  $\Delta[1/\rho_{xx}(B)]$  must be described by Eq. (11) over this magnetic field range and the value of phase-breaking time can be determined from the fitting procedure. The low-field magnetoresistance for structure 2 is presented in Fig. 4. Detailed analysis of the dependences  $\Delta[1/\rho_{xx}(B)]$  shows that the fitting values of prefactor  $\alpha$  do not depend on the temperature but to some extent de-

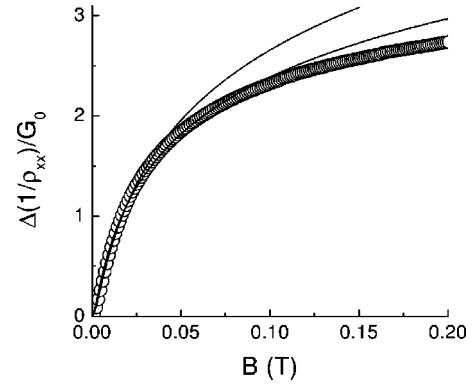


FIG. 4. The magnetic field dependence of  $\Delta[1/\rho_{xx}(B)]$  for structure 2,  $T=0.45$  K. Symbols are the experimental data. Lines are best fit to Eq. (11) made over different magnetic field ranges:  $\Delta B = (0-0.1)B_{tr}$  (upper line) and  $\Delta B = (0-0.3)B_{tr}$  (lower line).

pend on the fitting range  $\Delta B$ :  $\alpha = 1.2$  at  $\Delta B = (0-0.1)B_{tr}$  and  $\alpha = 0.9$  at  $\Delta B = (0-0.3)B_{tr}$ . The fitting values of  $\tau_\phi$  change with the fitting interval also and these changes are of the same order of magnitude as ones for  $\alpha$ . One can think that it results from the use of Eq. (11) obtained within the diffusion approximation. However, fitting by Eq. (20) from Ref. 15 obtained beyond the diffusion approximation does not give better accordance.

If one plots the temperature dependence of  $\tau_\phi$  found from the fixed fitting interval, we obtain it being close to  $T^{-p}$  with  $p \approx 0.85$  (Fig. 5), which is slightly below the theoretical value  $p = 1$  [see Eq. (4)].<sup>20</sup> The values of  $p$  for other structures are listed in Table I. Notice that the experimental values of  $\tau_\phi$  are close to theoretical ones (see Fig. 5). It should be mentioned that close to linear  $\tau_\phi$ -vs- $1/T$  dependence was observed in most papers but the magnitude often happened less the theoretical ones as much as 3–5 times. The reasons for that are unclear and one ought to suppose that additional phase-breaking mechanisms with close temperature dependence are essential in such structures. Therefore, the quantitative results for quantum correction, obtained for such structures seems to be inconclusive.

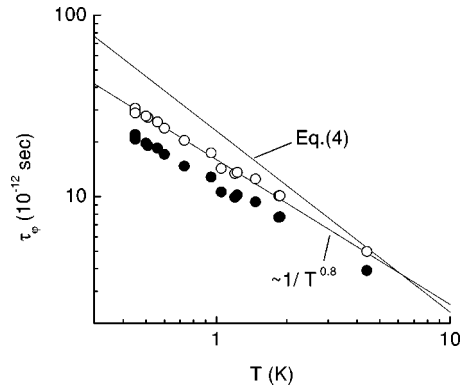


FIG. 5. Temperature dependence of  $\tau_\phi$ . Symbols are the results of fitting of the experimental data to Eq. (11) for different fitting range  $\Delta B$ :  $\Delta B = (0-0.1)B_{tr}$  (full circles) and  $\Delta B = (0-0.3)B_{tr}$  (open circles).



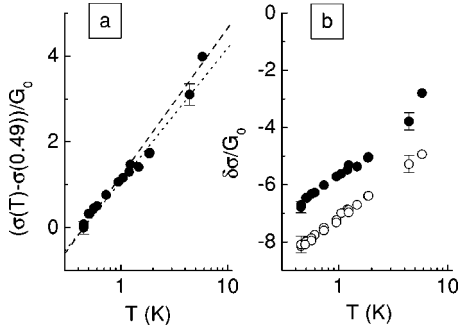


FIG. 6. (a) The dependence  $\Delta\sigma(T) = \sigma(T) - \sigma(T_0)$  with  $T_0 = 0.49$  K. Symbols are the experimental data, lines are given by Eq. (12) with  $(1 + 3/4\lambda) = 0.5$  and  $p = 1$  (dash line),  $p = 0.85$  (dotted line). (b) The temperature dependence of absolute value of total quantum correction to the conductivity determined by different ways (see text).

### VI. TEMPERATURE DEPENDENCE OF THE CONDUCTIVITY AT $B=0$ . ABSOLUTE VALUE OF THE QUANTUM CORRECTIONS

We turn now to analysis of the temperature dependence of the conductivity at  $B=0$  (Fig. 6). It is determined by the temperature dependence of both the interference correction and correction due to the EEI. As seen from Eqs. (3) and (4), the variation of  $\sigma$  with temperature has to decrease logarithmically with temperature decrease

$$\Delta\sigma(T) = \sigma(T) - \sigma(T_0) = G_0 \left[ p \ln\left(\frac{T}{T_0}\right) + \left(1 + \frac{3}{4}\lambda\right) \ln\left(\frac{T}{T_0}\right) \right], \quad (12)$$

where  $T_0$  is some arbitrary temperature. Thus, the slope in  $\Delta\sigma$ -vs- $\ln T$  dependence has to be equal to  $G_0[p + (1 + 3/4\lambda)]$ . Lines in Fig. 6(a) show the dependences  $\Delta\sigma(T)$  calculated from Eq. (12) with  $(1 + 3/4\lambda) = 0.52$  determined above (see Sec. IV) and with two values of  $p$ :  $p = 1$  obtained theoretically [see Eq. (4)], and  $p = 0.85$  describing the experimental  $\tau_\varphi$ -vs- $T$  dependence (see Fig. 5). It is evident that within experimental error the experimental results coincide with both dependences.

Now let us consider the absolute value of the total quantum correction  $\delta\sigma$ . On the one hand, we can find it from Eq. (3), using the parameters  $\tau_\varphi$  and  $(1 + 3/4\lambda)$  determined above, and  $\tau = \mu m/e$ , where  $m = 0.06 m_0$  is the electron effective mass in  $\text{In}_{0.15}\text{Ga}_{0.85}\text{As}$  quantum well. This values as function of temperature are plotted in Fig. 6(b) with open circles. The error shown in the figure is mainly determined by the difference in  $\tau_\varphi$  obtained for the different magnetic field intervals used for the fit. On the other hand, the absolute value of the quantum corrections at given  $T$  is equal to the difference between  $\sigma(T)$  and the Drude conductivity  $\delta\sigma(T) = \sigma(T) - \sigma_0$ . This value obtained with  $1/\rho_{xx}(B_{cr})$  as  $\sigma_0$  is represented by solid circles. As is seen these plots are parallel to each other, but differ by the value about  $(1.3 \pm 0.3)G_0$ .

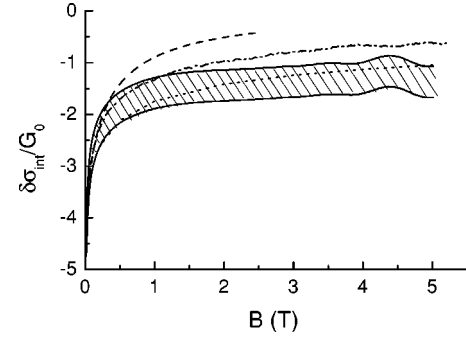


FIG. 7. The magnetic field dependence of the interference quantum correction  $\delta\sigma_{\text{int}}$  over the entire magnetic field range for structure 2,  $T = 1.5$  K. The shadowed area is the experimental result, spread is caused by error in determination of  $\sigma_0$  (see Sec. VI). The dashed line is the result of the diffusion approximation given by Eq. (11), the dot-dashed line takes into account both the back-scattering and non-back-scattering processes, the dotted line represents only the back-scattering contribution.<sup>17</sup>

What is the reason for noticeable difference between the absolute values of quantum correction, obtained by different ways? When evaluating the Drude conductivity we supposed that at  $B_{cr}$  the interference contribution was fully suppressed by magnetic field. In fact this correction does not equal to zero even at  $B_{cr} \gg B_{tr}$ , because at  $B > B_{tr}$  it decreases with increasing magnetic field very slowly.<sup>14,15,17,19</sup> Therefore, it is natural to associate the difference in Fig. 6(b) with residual interference contribution to the conductivity at  $B = B_{cr}$ . Thus, the proper values of the total quantum correction are represented in Fig. 6(b) by open circles, and the Drude conductivity should be more correctly estimated as  $\sigma_0 \approx \rho_{xx}^{-1}(B_{cr}) + (1.3 \pm 0.3)G_0$  (see Table I). Note, that the presence of some interference correction at large magnetic field does not affect the determination of  $(1 + 3/4\lambda)$  in Sec. IV because at  $B \gg B_{tr}$  the interference correction is practically temperature independent.

After we have found the Drude conductivity and the EEI contribution, we can obtain the interference correction to the conductivity over entire magnetic field range as

$$\delta\sigma_{\text{int}}(B) = \frac{(\sigma_{xx} - \Delta\sigma_{xx}^{ee})^2 + \sigma_{xy}^2}{\sigma_{xx} - \Delta\sigma_{xx}^{ee}} - \sigma_0. \quad (13)$$

In Fig. 7 the magnetic field dependence of  $\delta\sigma_{\text{int}}$  is presented together with theoretical dependences. One can see that  $\delta\sigma_{\text{int}}(B)$  calculated from Eq. (11) as

$$\delta\sigma_{\text{int}}(B) = \Delta[1/\rho_{xx}^{\text{int}}(B)] - \Delta[1/\rho_{xx}^{\text{int}}(\infty)]$$

well describes the magnetoresistance at low magnetic field (see also Fig. 4), but significantly deviates at  $B > 0.1$  T. It is not surprising because Eq. (11) was obtained within the diffusion approximation which is valid at  $B < B_{tr}$  ( $B_{tr} = 0.21$  T for this structure). The theoretical dependences  $\delta\sigma_{\text{int}}(B)$  obtained beyond the diffusion approximation for back-scattering processes and those taking into account non-back-scattering processes<sup>17</sup> are presented also. One can see that the experimental data lie closely to the curves obtained

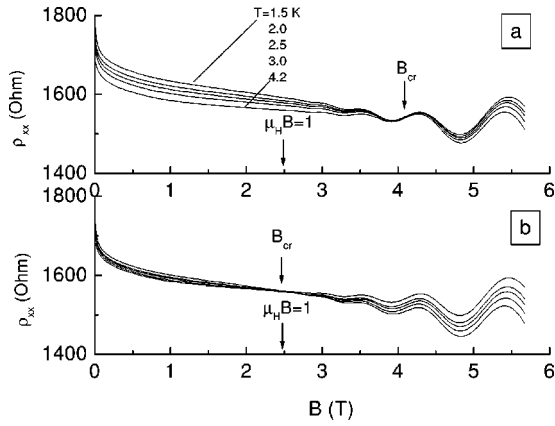


FIG. 8. Magnetic field dependences of  $\rho_{xx}$  at different temperatures as they have been measured (a), and those after extraction of the temperature dependence of mobility (b), structure 4.

beyond the diffusion approximation. However, our results do not allow to judge the role of non-back-scattering processes.

## VII. DISCUSSION

As shown in previous sections, all the temperature and magnetic field dependences for structure 2 are consistently described by the theory of quantum corrections. Namely, (i) for  $B \gg B_{tr}$ , the temperature dependence of  $\sigma_{xx}$  is logarithmic, whereas the temperature dependence of  $\sigma_{xy}$  is negligible, (ii) the low-field magnetoresistance is well described by the weak-localization theory with the value and temperature dependence of  $\tau_\varphi$  close to the theoretical ones, and (iii) the temperature dependence of the conductivity at  $B=0$  is logarithmic and quantitatively described by the sum of the interference and EEI contributions determined experimentally from the analysis of low and high magnetic field magnetoresistance, respectively.

Up to now we analyzed the results obtained for structure 2. Thorough analysis of experimental results for other structures shows that similar accordance with the theoretical predictions takes place for structures 1 and 3. It allows one to determine the EEI contribution to the conductivity and the values of  $(1 + 3/4\lambda)$  for different  $2k_F/K$  values (see Fig. 1). The results for these three structures are seen to fall on the curve, which is close in shape to the theoretical one, but lies somewhat lower. As we have emphasized above the results of other authors show large scatter (Fig. 1). To understand a possible reason for that let us consider the results for structure 4.

Structure 4, similar to structure 3 has  $\delta$ -doped barriers, but electron density is sufficiently higher in it (see Table I). At the first sight the magnetic field dependences of  $\rho_{xx}$  and  $\rho_{xy}$  at different temperatures [see Fig. 8(a)] are similar to those for structure 2 (Fig. 2). However, unlike structure 2, the value of  $B_{cr}$  is much greater than  $1/\mu$  (see Table I). Moreover, the nondiagonal component of conductivity tensor  $\sigma_{xy}$  significantly depends on the temperature at high magnetic field (Fig. 9) that is in conflict with the theoretical prediction for the EEI correction. This notwithstanding if one uses the slope of  $\sigma_{xx}$ -vs- $\ln T$  dependence at high magnetic

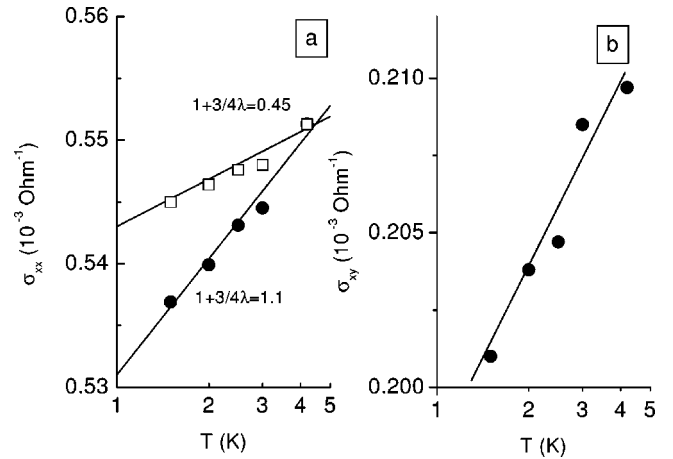


FIG. 9.  $\sigma_{xx}$  (a),  $\sigma_{xy}$  (b) as a function of temperature for structure 4,  $B = 10B_{tr}$ . Solid circles represent the results measured experimentally, open circles are  $\sigma_{xx}$  after extraction of the temperature dependence of mobility. The lines in (a) are given by Eq. (1), the line in (b) is the guide for an eye.

field for evaluation of the EEI contribution, we obtain the value of  $(1 + 3/4\lambda)$  about 1.1 [Fig. 9(a)]. As seen from Table I it is much greater than values of  $(1 + 3/4\lambda)$  for other structures. Furthermore, the temperature dependence of  $\sigma$  at  $B = 0$  for structure 4 remains logarithmic, but its slope gives the value of  $p + (1 + 3/4\lambda)$  about 2.4 instead of 1.25–1.5 obtained for structures 1–3 (see Table I).

The temperature dependence of  $\sigma_{xy}$  at high magnetic field in structure 4 seems to be incomprehensible. The interference correction should not depend on the temperature at  $B \gtrsim 10B_{tr}$ . The EEI should not affect  $\sigma_{xy}$ . Finally, the classical part of  $\sigma_{xy}$  is temperature independent at such strong degeneracy of electron gas ( $E_F/kT > 100$ ).

The contradictions are resolved, if one supposes that some fraction of the electrons in structure 4 occupies the states in  $\delta$ -doped layers unlike structure 3 with lower doping level. The density of these electrons is low enough and their selves do not carry charge through the sample, that follows from analysis of  $\rho_{xx}(B)$  and  $\rho_{xy}(B)$  dependences in the framework of model of two types of carriers. However, when temperature changes, the redistribution of these electrons within the  $\delta$  layers can lead to the temperature dependent disorder and, hence, to the temperature dependence of the mobility of the electrons in the quantum well. Estimations show that as low as 1% increase of the mobility with temperature increase from 1.5 to 4.2 K is enough to cause the temperature dependence of  $\sigma_{xy}$  observed experimentally. If we extract this 1% changing from  $\sigma_{xx}$  and  $\sigma_{xy}$ , all the results for this structure will be in accordance with the theoretical predictions, as for structures 1–3: the value of  $B_{cr}$  will coincide with  $1/\mu$  [Fig. 8(b)], the value of  $(1 + 3/4\lambda)$  will be equal to  $0.45 \pm 0.05$  [Fig. 9(a)], and the slope in the temperature dependence of  $\sigma$  at  $B=0$  will be equal to  $1.5 \pm 0.2$ .

It is worth noting that the temperature independence of  $\sigma_{xy}$  in high magnetic field was demonstrated only in Ref. 11, and as seen in Fig. 1 these results accord well with our data.

Thus, the results for structure 4 show that existence of carriers in doped layers can bring about an additional tem-

perature dependence of the mobility. This dependence should be taken into account when the parameters of the electron-electron interaction are determined in such type of structures. From our point of view, disregard of this fact can be one of the reasons for large scatter of results obtained by other authors.

### VIII. CONCLUSION

We have studied the quantum corrections to the conductivity in low-mobility 2D systems. The heterostructures with doped quantum well and doped barriers have been investigated. For the structures where electrons occupy the states in the quantum well only, successive analysis of the experimental data shows that all the results are self-consistently de-

scribed by the theory of quantum corrections. This allows us to determine reliably the value of the EEI contribution to the conductivity and its  $k_F$  dependence. It has been shown that the existence of carriers in doped layers when they are arranged in barriers can lead to the temperature dependent mobility even at low temperature when the electron gas in quantum well is strongly degenerated.

### ACKNOWLEDGMENTS

This work was supported in part by the RFBR through Grants No. 00-02-16215 and No. 01-02-17003, the Program *University of Russia* through Grants No. 990409 and No. 990425, the Federal Program *Physics of Solid-State Nanostructures*, and the CRDF through Grant No. REC-005.

- 
- <sup>1</sup>B.L. Altshuler and A.G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, edited by A.L. Efros and M. Pollak (North Holland, Amsterdam, 1985), p. 1.
- <sup>2</sup>P.A. Lee and T.V. Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985); G. Bergman, *Phys. Rep.* **107**, 1 (1984).
- <sup>3</sup>S.V. Kravchenko, G.V. Kravchenko, J.E. Furneaux, V.M. Pudalov, and M. D'Iorio, *Phys. Rev. B* **50**, 8039 (1994); S.V. Kravchenko, W.E. Mason, G.E. Bower, J.E. Furneaux, V.M. Pudalov, and M. D'Iorio, *ibid.* **51**, 7038 (1995).
- <sup>4</sup>B.L. Altshuler and A.G. Aronov, *Zh. Éksp. Teor. Fiz.* **77**, 2028 (1979) [*Sov. Phys. JETP* **50**, 968 (1979)].
- <sup>5</sup>A.M. Finkelstein, *Zh. Éksp. Teor. Fiz.* **84**, 168 (1983) [*Sov. Phys. JETP* **57**, 97 (1983)].
- <sup>6</sup>S. Hikami, A. Larkin, and Y. Nagaoka, *Prog. Theor. Phys.* **63**, 707 (1980).
- <sup>7</sup>B.L. Altshuler, A.G. Aronov, A.I. Larkin, and D.E. Khmel'nitskii, *Zh. Eksp. Teor. Fiz.* **81**, 788 (1981).
- <sup>8</sup>B.J.F. Lin, M.A. Paalanen, A.C. Gossard, and D.C. Tsui, *Phys. Rev. B* **29**, 927 (1984).
- <sup>9</sup>I.G. Savel'ev, T.A. Polyanskaya, and Yu.V. Shmartsev, *Fiz. Tekh. Poluprovodn. (S.-Peterburg)* **21**, 2096 (1987).
- <sup>10</sup>W. Poirier, D. Mailly, and M. Sanquer, *Phys. Rev. B* **57**, 3710 (1998).
- <sup>11</sup>S.S. Murzin, *Pis'ma Zh. Eksp. Teor. Fiz.* **67**, 201 (1998).
- <sup>12</sup>B.L. Altshuler, D.L. Maslov, and V.M. Pudalov, *Phys. Status Solidi B* **218**, 193 (2000).
- <sup>13</sup>Recently, the paper on classical mechanism of negative magnetoresistance in two dimensions has appeared [Alexander Dmitriev, Michel Dyakonov, and Remi Jullien, cond-mat/0103490 (unpublished)]. Estimations for our structures show that the negative magnetoresistance due to this mechanism is no more than  $(0.3-0.4)G_0$  at magnetic field  $B=\mu^{-1}$ . As a result  $\rho_{xx}(B_{cr})$  differs from the Drude conductivity on the same value. It is within the limits of our experimental error. Moreover, this mechanism is temperature independent, therefore, taking it into account does not change the values of parameters which are obtained from the temperature dependences.
- <sup>14</sup>S. Chakravarty and A. Schmid, *Phys. Rep.* **140**, 193 (1986).
- <sup>15</sup>H.-P. Wittman and A. Schmid, *J. Low Temp. Phys.* **69**, 131 (1987).
- <sup>16</sup>M.I. Dyakonov, *Solid State Commun.* **92**, 711 (1994).
- <sup>17</sup>I.V. Gorniy, A.P. Dmitriev, and V.Yu. Kachorovskii, *Phys. Rev. B* **56**, 9910 (1997).
- <sup>18</sup>A.G. Groshev and S.G. Novokshenov, *Fiz. Tverd. Tela (Leningrad)* **42**, 1322 (2000); A.G. Groshev, S.G. Novokshenov, and I.V. Gorniy, *ibid.* **43**, 766 (2001).
- <sup>19</sup>G.M. Minkov, A.V. Germanenko, O.E. Rut, and I.V. Gorniy, *Phys. Rev. B* **61**, 13 164 (2000).
- <sup>20</sup>From our point of view some distinction of the power  $p$  from unity does not demonstrate the failure of Eq. (4). The fact is that the experimental values of  $\tau_\varphi$  are found as fitting parameters and they are different for the different fitting intervals. It is natural to suppose that an error in determination of  $\tau_\varphi$  is about this difference (see Fig. 5). From this reasoning one can conclude that  $p=1$  does not contradict the experimental data as well.