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Subwavelength imaging with opaque nonlinear left-handed lenses

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We consider a slab of a composite left-handed metamaterial with quadratic nonlinear response and discuss the properties of opaque nonlinear left-handed lens. We find the conditions when this left-handed flat lens can create, with a subwavelength resolution, an image of the second-harmonic field of the source being opaque for the fundamental frequency. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2034114]

One of the intriguing properties of the recently demonstrated left-handed metamaterials, i.e., composite materials with simultaneously negative real parts of dielectric permittivity and magnetic permeability,[1] is their ability to focus electromagnetic waves by a flat slab of the material, the property which makes these materials quite different from the conventional optical lenses with the positive refractive index needed to have curved surfaces to form an image. Recently, Pendry[2] argued that a slab of a lossless left-handed material with $\epsilon=\mu=-1$, where ϵ is dielectric permittivity and μ is magnetic permeability should behave like *a perfect lens* enabling to obtain an ideal image of a point source through the amplification of the evanescent components of the field. While recent experimental demonstrations confirmed the main features of negative refraction of the left-handed materials,[3,4] near-perfect imaging by a flat lens and near-field focusing are severely constrained because of strong dissipation and dispersion of metamaterials. Nevertheless, numerical studies indicate[5] that nearly-perfect imaging should be expected even under realistic conditions when both dispersion and losses are taken into account.

Most of the properties of left-handed metamaterials have been studied only for linear waves. However, it has been already noticed that the left-handed metamaterials may possess quite complicated nonlinear magnetic response,[6,7] their properties can be altered by inserting diodes in the split-ring resonators,[8] and nonlinear materials can demonstrate interesting features in the second-harmonic generation.[9] In this Letter, we analyze the imaging properties of a slab of metamaterial with quadratic nonlinear response and demonstrate, both analytically and numerically, that such a flat lens can form an image of the second-harmonic field of the source being opaque at the fundamental frequency.

We consider a slab of a left-handed metamaterial (see Fig. 1) with the thickness D, and assume that this slab is a three-dimensional composite structure made of wires and split-ring resonators (SRRs) in the form of a cubic lattice. When the lattice period d is much smaller than the radiation wavelength λ ($d \ll \lambda$), this composite structure can be described in the framework of the effective-medium approximation, and it can be characterized by effective values of dielectric permittivity and magnetic permeability. For the specific structure and monochromatic waves [$\sim\exp(i\omega t)$] these dependencies can be written as follows:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - i\gamma_e)}, \quad \mu(\omega) = 1 + \frac{F\omega^2}{\omega_0^2 - \omega^2 + i\gamma_m\omega}, \tag{1}$$

where ω_p is the effective plasma frequency, $\omega_0=\bar{\omega}_0\sqrt{(1-F)}$, $\bar{\omega}_0$ is the SRR eigenfrequency, F is the filling fraction of SRRs, γ_e and γ_m are the corresponding damping coefficients, ω is the frequency of the external electromagnetic field. In the frequency range where the real parts of ϵ and μ are both negative and for $\gamma_e, \gamma_m \ll \omega$, such a composite structure demonstrates left-handed-medium transmission properties, whereas for $\omega < \omega_0$, the slab is opaque because the signs of ϵ and μ are opposite.

In order to introduce quadratic nonlinearity into this structure, we assume that each SRR includes a nonlinear element, e.g., a diode inserted into the SRR slot.[8] Each diode has an asymmetric current-voltage characteristics. As a re-

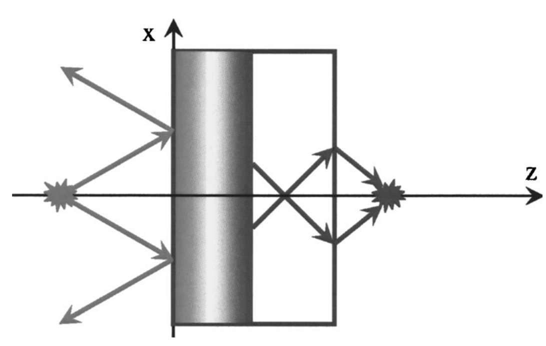

FIG. 1. Schematic of the problem. Electromagnetic waves emitted by a source at $z=-z_s$ are reflected from an opaque metamaterial slab. Inside the slab, the decaying field at the frequency ω generates the second harmonics at 2ω, which penetrates through the slab creating an image.

sult, the unit cell does not possess a center of symmetry, i.e., the amplitudes of the magnetic momenta induced in the unit cell by the magnetic field **H** and **−H** are different, and the response of such an effective quadratic nonlinear medium should include the second-harmonic field.

Our idea is to satisfy the well-known conditions of Pendry's perfect lens at the frequency 2ω assuming $\mu(2\omega)=\epsilon(2\omega)=-1$, and derive the corresponding conditions for the fundamental field,

$$\epsilon(\omega) = -7, \quad \mu(\omega) = \frac{(3-F)}{(3-2F)}. \tag{2}$$

For this choice of the material parameters, a metamaterial is opaque at the fundamental frequency ω, and the waves do not penetrate into the slab. However, an effective nonlinear quadratic response of the metamaterial allows the process of the second-harmonic generation. For the conditions (2) the metamaterial with the dispersion (1) is transparent at the frequency 2ω, and we expect that the second-harmonic field can propagate through the slab creating an image of the source behind the slab.

Using the so-called undepleted pump approximation,[10] we obtain the equation for the TM-polarized second-harmonic field $H_y^{(2\omega)}(x,z)$ inside the slab

$$\Delta H_y^{(2\omega)} + K^2(2\omega)H_y^{(2\omega)} = -16\pi\frac{\omega^2}{c^2}\epsilon(2\omega)M_{\mathrm{NL}}^{(2\omega)}, \tag{3}$$

where Δ is the Laplacian acting in the space (x,z), $K^2(2\omega)=4k_0^2\epsilon(2\omega)\mu(2\omega)$, $k_0=\omega/c$, and $M_{\mathrm{NL}}^{(2\omega)}$ is the nonlinear magnetization of the unit volume of the metamaterial at the frequency 2ω, which appears due to the nonlinear magnetic momentum of SRR, $M_{\mathrm{NL}}^{(2\omega)}=-[3\chi(\omega)/4][H_y^{(\omega)}(x,z)]^2$, where

$$\chi(\omega) = \frac{(\pi a^2)^3\omega_0^4}{c^3d^3U_cR_d\omega^2}\left[\left(1-\frac{\omega_0^2}{\omega^2}\right)^2 - \frac{i\gamma_m}{\omega}\left(2+\frac{\omega_0^2}{\omega^2}\right)\right]^{-1}.$$

Here $H_y^{(\omega)}(x,z)$ is the spatial distribution of the magnetic field at the frequency ω in the slab, a is the radius of the resonator rings, R_d is the differential resistance of the diode at zero voltage, and U_c is the diode parameter defined from the current-voltage characteristics of the diode taken in the form $I=I_0[\exp(U/U_c)-1]$. The right-hand side of Eq. (3) vanishes outside the metamaterial slab.

Applying the Fourier transform along the x direction, we obtain the equation for the function $\bar{H}_y^{(2\omega)}=\bar{H}_y^{(2\omega)}(k_x,z)$ in terms of $\bar{H}_y^{(\omega)}=\bar{H}_y^{(\omega)}(k_x,z)$ and $G=G(k_x,z)$ which are the Fourier transforms of $H^{(\omega)}(x,z)$, $H^{(2\omega)}(x,z)$, and $[H^{(\omega)}(x,z)]^2$, respectively,

$$\frac{d^2\bar{H}_y^{(2\omega)}}{dz^2} + [K^2(2\omega)-4k_x^2]\bar{H}_y^{(2\omega)} = \eta G, \tag{4}$$

where $\eta=12\pi k_0^2\epsilon(2\omega)\chi(\omega)$. Using the convolution theorem, we express the function G through the spectrum of the magnetic field at the fundamental frequency,

$$G = \int_{-\infty}^{\infty}\bar{H}^{(\omega)}(k_x',z)\bar{H}^{(\omega)}(k_x-k_x',z)dk_x'. \tag{5}$$

In the undepleted pump approximation, the function $\bar{H}^{(\omega)}(k_x,z)$ can be found as a solution of the linear problem for the electromagnetic field at the fundamental frequency transmitted into the left-handed slab,

$$\bar{H}^{(\omega)} = \frac{2\kappa_1 e^{-ik_0\kappa_1 z_s}}{D(\omega,k_x)}[Z_1e^{k_0\kappa_2 z} - Z_2e^{k_0\kappa_2(2D-z)}]S(\gamma), \tag{6}$$

where $S(\gamma)$ is the spectral function of the source at the fundamental frequency located at the distance z_s from the left-handed slab, $\kappa_1=\sqrt{1-\gamma^2}$, $\kappa_2=\sqrt{\gamma^2-\epsilon(\omega)\mu(\omega)}$, $\gamma=k_x/k_0$, $Z_{1,2}=\kappa_1\pm i\kappa_2/\epsilon(\omega)$, and $D(\omega,k_x)=Z_1^2-Z_2^2\exp(2k_0\kappa_2D)$. The spectral function $S(\gamma)$ includes both fast propagating ($\gamma\leqslant 1$) and slow evanescent ($\gamma>1$) spatial harmonics. Possible distortions of the second-harmonic image can be caused by the pre-exponential factor in Eq. (6), and the main effect is due to the pole singularity defined by the equation $D(\omega,k_x)=0$, that characterizes the resonant excitation of *surface polaritons*. In the case of a thick slab, i.e. when $k_0\kappa_2D\gg 1$, the corresponding resonant wave number of surface waves can be found in the form

$$\gamma_{sp}^2 = \frac{\epsilon(\omega)[\epsilon(\omega)-\mu(\omega)]}{\epsilon^2(\omega)-1}. \tag{7}$$

Substituting the explicit expressions for $\epsilon(\omega)$ and $\mu(\omega)$ from Eq. (2) into Eq. (7), we obtain a simple estimate for an expected resolution limit of the nonlinear left-handed lens in terms of the critical (limiting) wave number, $\gamma_{\lim}^2\approx 1+(3-F)/7(3-2F)$. However, the existence of this critical wave number does not necessarily limit the lens resolution and, in reality, the effect produced by surface waves on the imaging characteristics of the nonlinear lens depends on the efficiency of the mode excitation by each particular source.

Analytical solution of the problem for the spatial spectrum of the second-harmonic field transmitted through the left-handed slab can easily be obtained for a narrow spectrum of the source, i.e., when the width of the source spectrum at the fundamental frequency does not exceed the value γ_c, where $\gamma_c^2\approx|\epsilon(\omega)\mu(\omega)|$. Then, we can use the impedance boundary conditions for the fundamental field at the interface between vacuum and the metamaterial slab at $z=0$. Subsequently employed numerical simulations suggest that this approximation remains valid provided $\gamma_c\gg\gamma_{\lim}$.

To solve the problem analytically, we assume that the wave at the fundamental frequency ω penetrates inside the slab on a distance (the skin layer) much smaller than the slab thickness D, i.e., $D\delta\gg 1$, where $\delta=k_0\sqrt{-\epsilon(\omega)\mu(\omega)}$. Taking into account the expressions for $\epsilon(\omega)$ and $\mu(\omega)$ from Eq. (2), we note that the penetration depth for the fundamental frequency, $\sim(2\pi\delta)^{-1}$, does not exceed $\lambda/17$. Then, Eq. (4) can be solved analytically for the second harmonic of the magnetic field behind the slab (for $z>D$),

$$\bar{H}_y^{(2\omega)}(\gamma,z) = -\frac{1}{2}\left\{1-i\frac{\kappa_2\epsilon(\omega)}{\kappa_1\epsilon(2\omega)}\right\}Ce^{2k_0\kappa_1(2D-z)}. \tag{8}$$

Here, $C=\eta A_0(\gamma)/4k_0^2[\epsilon(2\omega)\mu(2\omega)-\epsilon(\omega)\mu(\omega)]$, $A_0(\gamma)=\int_{-\infty}^{\infty}\xi(\gamma')\xi(\gamma-\gamma')d\gamma'$, and $\xi(\gamma)=e^{\delta z}\bar{H}^{(\omega)}(\gamma,z)$ does not depend on z.

Thus, the squared field at the fundamental frequency acts as an effective source for generating the second harmonic and, as a result, the image of the squared field is reproduced by the nonlinear left-handed lens. For the wide beam with

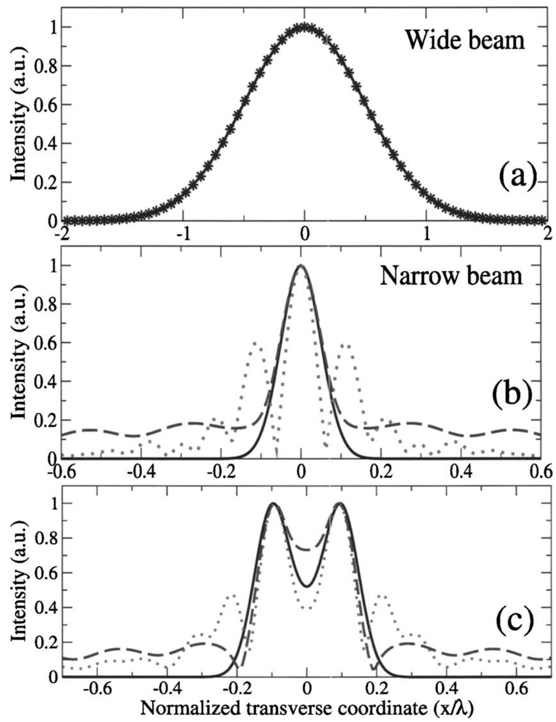

FIG. 2. Numerical results for imaging by a nonlinear left-handed lens. Shown are the intensities of the fundamental field at the source location (solid) and the second-harmonic field at the image plane for different cases (stars, dashed, and dotted lines). (a) Wide beam (the width is λ) generated by a single source, $D=\lambda$, $z_s=\lambda/2$. (b) Narrow beam (the width is $\lambda/10$) generated by a single source; dashed, the image for $D=\lambda/10$, $z_s=\lambda/20$; dotted, the image for $D=0.3\ \lambda$, $z_s=0.15\ \lambda$. (c) Imaging by two sources separated by the distance $\lambda/5$; dashed, the image for $D=\lambda/10$ and $z_s=0.03\ \lambda$; dotted, the image for $D=0.3\ \lambda$ and $z_s=\lambda/5$.

narrow spectra it can be further shown that this image appears at the point $z_{\text{im}}=2D-z_s$, this result coincides with the corresponding result for the linear lens.[2]

For the case when the size of the source is comparable or less than the wavelength λ of the fundamental-frequency wave, Eq. (3) is solved numerically. In Figs. 2(a)–2(c), we present our numerical results for the intensity distribution of the incident beam at the source point and the field distribution of the second-harmonic beam at the image location, normalized to the field maxima. The actual amplitude of the electromagnetic field at the image location is lower than the amplitude of the source because of a finite efficiency of the process of the second-harmonic generation. For the objects with the spatial scale larger or equal to the radiation wavelength, the second-harmonic field profile coincides with the intensity of the fundamental field generated by the source, as shown in Fig. 2(a).

However, when the source contains the spatial scales less than the radiation wavelength, the imaging properties of the nonlinear lens depend strongly on the slab thickness D. As an example, in Fig. 2(b) we show the results for the transmission of an incident Gaussian beam of the width $\lambda/10$ which reproduces almost exactly the source profile at the image plane in the case of a thin lens (dashed line) but generates a strongly distorted image when the slab thickness becomes larger than a half of the wavelength λ. Distortions appear as periodic variation of the second-harmonic field being caused by excitation of surface waves.

Figure 2(c) shows the numerical results for imaging of two sources that generate the Gaussian beams with the maxima separated in the transverse direction by $\lambda/5$. Again, the image reproduces very well the source for a thinner lens, and therefore a thin nonlinear lens does provide a subwavelength resolution of the second-harmonic field. In contrast to the linear flat lens, the resolution of the nonlinear lens depends on the distance z_s between the source and the lens, and the optimal distance can be determined separately for each particular case.

In conclusion, we have studied the second-harmonic generation in a slab of left-handed metamaterial with quadratic nonlinear response. We have demonstrated that such a slab can act as a nonlinear lens and it can form an image of the second-harmonic field of the source being opaque at the fundamental frequency, with the resolution that can be made better than the radiation wavelength.

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